

Index

- A_n is simple, 15
- E_f , 137
- G -equivalence, 29
- G -equivariant, 28, 84
- G -invariant, 26
- G -invariant subspace, 86
- G -set, 22
- K^H , 161
- $[g]$, 3
- $[k : \mathbb{F}]$, 131
- $\text{Cf}(G)$, 104
- $\text{Cl}(G)$, 3
- $\text{Fr}: k \rightarrow k$, 145
- $\text{Gal}(K/k)$, 153
- $\text{Gl}(K/k)$, 208
- $\text{Gl}_n(\mathbb{F})$, 55
- $\text{Hom}(k, K)$, 152
- $\text{Hom}_G(V, W)$, 101
- $\text{Ind}_H^G \rho$, 118
- $\text{Irr}(G)$, 104
- $\Phi_d(x)$
 - cyclotomic polynomial, 171
- $\text{Quot}(R)$, 129
- $\text{Res}_H \rho$, 115
- $\Sigma(X)$, 21
- Σ_n , 21
- St_p , 192
- $\delta_H(g)$, 119
- $\binom{G}{k}$, 23
- $\mathbb{F}(S)$, 132
- $\mathbb{F}(a_1, \dots, a_n)$, 132
- \mathbb{F}_p , 131
- Doc^+ , 19
- $\phi(x)$, 172
- k -transitive action, 32
- $m_\alpha(x)$, 134
- n th roots of unity, 171
- p -group, 30
- $p(n)$, 5
- $\mathcal{I}(K, k)$, 161
- \mathcal{T}_G , 161
- abelianization, 47
- action of a group on a set, 22
- adjoining a set to a field, 132
- adjoint mapping, 62
- algebra, 215
- algebraic closure, 148
- algebraic extension, 135
- algebraic integer, 98
- algebraic number, 98, 135
- algebraically closed, 148
- alternating group, 4
- Artin, E., 148
- automorphism, 145
- average, 87
- basis, 51
- bilinear mapping, 61
- binomial coefficient, 37
- blocks, 217
- Brauer, R., 216
- Burnside's $p^\alpha q^\beta$ Theorem, 179
- Burnside's Orbit Counting Theorem, 28
- Burnside's Transfer Theorem, 43
- Burnside, W., 43, 77
- Cardano, G., 180
- Cauchy's Formula, 4
- Cauchy's Theorem, 36
- Cauchy, A., 4

- Cayley's Theorem, 29
 Cayley, A., 29
 center of a group, 3
 centralizer of a subgroup, 43
 centralizer of an element, 7
 change-of-basis matrix, 74
 character, 93
 irreducible, 101
 character table, 108
 characteristic
 p , 131
 zero, 131
 characteristic of a field, 87, 131
 characteristic polynomial, 66, 90
 Chebotarev Density Theorem, 204
 circle division, 170
 Class Equation, 30
 class function, 94
 Clifford's Theorem, 116
 Clifford, A.H., 116
 commutator, 17
 commutator subgroup, 17
 companion matrix, 66
 completely reducible, 88
 complex conjugation, 64
 composition factors, 11
 composition series, 11
 compositum, 169
 conjugacy classes, 3
 in A_n , in Σ_n , 8
 conjugate elements, 3
 conjugate fields, 167
 conjugate of a class function, 116
 conjugate of subgroup, 22
 conjugation by g , 2
 constructible number, 150
 content, 138
 coordinate mapping with respect to B , 55
 Correspondence Theorem, 9
 cubic resolvent, 195
 Curtis, C., 214
 cycle notation, 4
 cyclic vector, 211
 cyclotomic polynomial, 141, 171
 cyclotomy, 170

 Dedekind's Independence Theorem, 162
 Dedekind, R., 162, 172, 200
 degree of a representation, 80
 del Ferro, S., 180
 density, 204

 derivative, 142
 derived series, 17
 Deuring, M., 211
 Dickson, L., 216
 dihedral group, 6
 dimension, 52
 direct sum, 56
 discriminant, 192
 disjoint union, 27
 division algebra, 216
 Division Algorithm, 133
 dodecahedron, 19, 78
 dot product, 63
 doubling the cube, 149
 doubly transitive action, 33

 eigenvalue, eigenvector, 65
 Eisenstein, G., 140
 elementary symmetric polynomials, 188
 equivalent actions, 23
 equivalent representations, 84
 Euclidean domain, 133
 Euler ϕ -function, 172
 evaluation mapping, 134
 even permutation, 4
 exact sequence, 59
 extension, 131
 algebraic, 135
 finite, 131
 Galois, 166
 infinite, 131
 normal, 165
 radical, 181
 separable, 165
 simple, 133
 extension of Q by K , 14

 faithful action, 29
 fellow rower, 43
 Fermat primes, 176
 Ferrari resolvent, 195
 field, 48, 128
 field homomorphism, 130
 field of fractions, 129
 field of rational functions, 129
 finite-dimensional vector space, 51
 First Isomorphism Theorem, 1
 fixed element, 26
 fixed field, 161
 fixed set, 27
 fixed subspace, 82

- Fontana, N., 180
 Frobenius action, 32
 Frobenius automorphism, 145, 199
 Frobenius complement, 32
 Frobenius Density Theorem, 204
 Frobenius group, 32, 203
 Frobenius Reciprocity, 120
 Frobenius, F.G., 30, 120
 fundamental concepts, 1
 Fundamental Theorem of Algebra, 169
 Fundamental Theorem of Galois Theory, 167
 Fundamental Theorem of Symmetric Polynomials, 190
- Galois extension, 166
 Galois group, 153
 cubics, 195
 quadratics, 195
 Galois, É., 1, 19, 48, 128, 152, 199
 Gauss's Lemma, 138
 Gauss, C.-F., 48, 138, 173, 176
 general linear group, 55
 generators, 6
 Girard, A., 191
 Gram–Schmidt process, 65
 greatest common divisor, 138
 group determinant, 214
 group of automorphisms, 152
- Hasse diagram, 2
 Hawkins, T., 214
 Hermite, C., 98, 217
 Hermitian inner products, 64
 Hom-tensor interchange, 62
- image, 1, 54
 index of subgroup, 7
 indicator function, 95, 119
 inner automorphism, 2
 inseparable polynomial, 142
 integral domain, 129
 integral unit quaternions, 83
 intermediate field, 131
 internal direct sum, 57
 Inverse Galois Problem, 201
 inverse image, 9
 irreducible, 86
 irreducible element, 129
 isomorphism, 53
 Iwasawa's Lemma, 34, 72
 Iwasawa, K., 34
- Jordan–Hölder Theorem, 12
- kernel
 of a homomorphism, 1
 kernel of a character, 112
 kernel of a linear transformation, 54
 kernel of a representation, 81
 kernel of the action ρ , 29
 Klein, F., 217
 Kronecker product, 63
 Kronecker's Theorem, 137
 Kronecker, L., 137, 197, 201
 Kronecker–Weber Theorem, 201
- Lagrange resolvent, 182
 leading term, 188
 left action, 22
 Legendre symbol, 47
 lexicographic order, 188
 Lie algebra, 218
 Lie group, 218
 lift, 111
 Lindemann, F., 98
 linear transformation, 52
 linearly (in)dependent, 50
 Lucas, É., 37
- Möbius function, 146
 Maschke's Theorem, 87
 Maschke, H., 87
 maximal element, 51
 maximal ideal, 129
 maximal normal subgroup, 10
 maximal subgroup, 33
 minimal polynomial, 98, 134
 modular representation theory, 216
 module, 215
 Monster, x
 multiplicity, 142
- Newton's identity, 191
 Newton, I., 191
 Noether, E., 201, 211
 nondegenerate inner product, 64
 norm, 133, 212
 normal basis, 208
 Normal Basis Theorem, 209
 normal closure, 168
 normal complement, 43
 normal extension, 165
 normal series, 11

- normal subgroup, 1
- normalizer, 24
- null space, 54
- number of partitions of n , 5

- odd permutation, 4
- orbit, 24
- Orbit-Stabilizer Theorem, 25
- orthogonal complement, 65
- Orthogonality Theorem
 - First, 103
 - Second, 109

- partition of a number n , 5
- periods, 174
- permutation, 3
- positive definite inner product, 64
- presentation of a group, 6
- prime subfield, 131
- primitive n th root of unity, 171
- primitive element, 208
- Primitive Element Theorem, 208, 212
- primitive polynomial, 138
- primitive root, 144
- projective space, 71
- projective special linear group, 71

- quaternion group, 31
- quotient field, 129
- quotient vector space, 60

- radical extension, 181
- reducible, 86
- refinement, 18
- regular action, 32
- representation, 80
 - faithful, 81
 - induced, 118
 - linear, 113
 - permutation, 81, 96
 - regular, 81
 - trivial, 80
- resolvent of P , 193
- restriction, 22, 114

- Schreier Refinement Theorem, 18
- Schur's Lemma, 90
- Schur, I., 90
- Second Isomorphism Theorem, 2
- separable polynomial, 142
- set of fixed elements, 27

- short exact sequence, 14, 59
- similar matrices, 57, 66
- simple group, 10
- simple module, 216
- simple radical extension, 181
- simple tensor, 61
- solvable by radicals, 181
- solvable groups, 14
- Span, 50
- spanning set, 50
- special linear group, 67
- special orthogonal group, 218
- splitting field, 136
- squaring-the-circle, 98
- stabilizer subgroup, 24, 192
- stable, 86
- Steinitz, E., 148
- subfield, 128
- subgroup
 - normal, 1
 - subquotients, 11
- subrepresentation, 86
- subspace, 50
- Sylow p -subgroup, 36
- Sylow Theorems, 36
- Sylow, L., 36
- symmetric group on n letters, 3, 21
- symmetry group on a set X , 21

- tensor product, 60
- tensor product of matrices, 63
- tensor product of representations, 85
- the *the*, 158
- Third Isomorphism Theorem, 12
- trace, 93, 212
- transcendental number, 98, 135
- transfer, 42
- transitive action, 26
- transposition, 4
- transvections, 68
- transversal, 42, 108, 118
- trisecting an angle, 149
- trivial action, 24
- trivial extension, 14
- type of a permutations, 5

- unitary representations, 218
- universal example, 187

- vector space, 48
- Viergruppe, 11

Wantzel, P.-L., 177
warp and woof, 167
Weber resolvent, 205
Weber, H.M., 201, 205

Zassenhaus Lemma, 18
Zorn's Lemma, 51