The Cambridge Maths team and acknowledgements

The Cambridge Mathematics team is highly collaborative with a wide range of expertise. Here are those team members who have contributed in some way to this book, and to the *Espressos* which form a major part of it.

Lynn Fortin

is our humanities expert. She provided feedback on and proofread all the original *Espressos* and the text of this book, helpfully pointing out Lucy's degenerate tendencies to constantly split infinitives. Lynn also provided invaluable support on referencing and with copyright permissions.

Tabitha Gould

designs Cambridge Mathematics content in number, algebraic structure and professional development. From this perspective, and as a former teacher, she has provided feedback on drafts and has also co-written several *Espressos*.

Rachael Horsman

is our expert in geometry and writes Cambridge Mathematics content in geometry, professional development and teacher knowledge and beliefs. Along with the rest of the writing team, she provides feedback and has co-written several *Espressos*.

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Ray Knight

designs the *Espressos*, digitising the infographics and creating the final layouts of the PDFs, so he was well placed to undertake the design of this book.

Darren Macey

is a former maths teacher who writes Cambridge Mathematics content in statistics education, professional development and teacher pedagogical knowledge. He has provided feedback on *Espressos* and has co-written several, as well as contributing to the design of the infographics.

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Lynne McClure

was the originator of the idea for *Espressos* and has co-written several. As Director of the Cambridge Mathematics project, she does the final edits both on *Espressos* and on this book, and signs off on all our products.

Nicky Rushton

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Lucy Rycroft-Smith

has written or co-written all the *Espressos* in this book. She designed the original structure for *Espressos* and draws their draft infographics. Her research into evaluating research summaries in mathematics education has helped to shape *Espressos*, this book and ongoing professional development work at Cambridge Mathematics.

We sincerely thank all those who have collaborated with us on *Espressos*, and kindly offered feedback and suggestions for improvement.

We would also like to thank all those who have used *Espressos* to begin interesting conversations about mathematics teaching, education research, and mathematical concepts, and have contacted us to let us know. If you are using *Espressos*, please do tell us!

Introduction

This book is intended for anyone teaching maths to young children – that is, children aged approximately 0–12. It's both an exclusive and inclusive club, in that some quite extraordinary people belong to it (and that's you) – and yet it is open, friendly and warm to newcomers.

This warmth can be felt all over the world, in classrooms, staffrooms and in digital spaces. You only have to look at what happens when a new maths teacher asks a question on social media, usually prefaced with "Sorry if this is a stupid question, but…" Flocks of friendly maths teachers swoop to their rescue, often sharing their resources, their hard-won tips and their best practice, all whittled down over years of classroom handling to a smooth, user-friendly surface.

As part of our blog series *Seven questions with…*, we asked people who had made a name for themselves in mathematics education whether a good maths teacher was born or made. This was intentionally provocative, and many of them quite rightly told us off for it (Professor Gabriel Stylianides told us firmly and memorably, "I don't believe in this dichotomy"). But there has been a consensus among our interviewees over the years – they generally agree that a good maths teacher is made, although some qualities that may be innate can help. Bobby Seagull told us, "No-one comes out of the womb with a whiteboard marker and perfect teaching skills"; Professor Nira Chamberlain was crystal clear, stating, "Mathematicians evolve, develop and grow"; Jo Morgan told us, "I believe that the science of teaching can be learnt, and I'm doing everything I can to learn it." Professor Francis Su, writer of the beautiful book *Mathematics for Human Flourishing*, said, "I believe that EVERY person can improve how they teach mathematics"; Professor Marcia Burrell said, "A brilliant maths teacher is made – an absolute no to the 'born' possibility. I think once you delve deeply into those teachers who are good or great, you realize that they have been mentored and guided 'into' their brilliance."

What a beautiful phrase: *guided into their brilliance*. We believe this too. We know from research that maths teachers come in many guises, and they can all be successful; there is no single "right" way to teach maths, and no single "right" way to be a maths teacher. We have met incredible maths teachers who are loud, quiet, energetic, calm, funny, serious, neat, chaotic, methodical, creative, steady, unpredictable, and everything in between. We believe that guiding maths teachers into their brilliance involves working with who they are, not trying to make them someone else, and that this works in two distinct ways: providing stimulating ideas and removing barriers.

So this book is not about telling you how to teach; it will not extend into your classroom unless you choose to let it. It will provide the summarised research, the classroom implications and the invitations to reflect on them that have the potential to contribute towards your professional learning, but you provide the rest – the knowledge of your children, your classroom, your school and your community that will help you to decide where to take your professional development. You not only provide this knowledge of context, but a critical professional eye that takes it into account. You may find yourself, through using this book, saying "But that wouldn't work that way here, because…" or "That didn't work there, but it might work with this class, because…" or even "What a load of rubbish, they think it works like THAT?" This is crucial, and empowering. Research evidence is not a pill for teachers to swallow; it is a glorious buffet. Your professional agency matters, and we wholly encourage using research ideas critically, which means choosing whatever you want to put on your plate whenever you like, giving things a nibble, and reserving the right to dislike things that you have tried. As you move forward in your professional learning journey, you may critique not just the ideas in the research evidence but also the theories ("we think this works like that"), the methodologies ("we measured this like that") and the conclusions ("so we think this means that"). Researchers are not perfect, and they may fail to see important issues or implications from their work that you pick up on. Engaging with research isn't just about reading it, it's about how you respond to it – your thoughts matter. We envisage your use of this book as a springboard for those types of rich, critical discussions that inspire you to go back into the classroom with renewed energy and excitement. After each *Espresso*, you'll see a set of pages called *Expressions*, which invite you to respond critically to the research ideas in just this manner.

When we asked him about maths teaching, Eddie Woo, with the articulacy that has helped make him a global maths teaching phenomenon, said, "It has taken me years (and many mistakes!) to develop the craft of asking good questions, of being able to discern student misconceptions, and of having a broad understanding of the interconnected areas of mathematics to connect concepts together in coherent ways." Dr Cornelia Connolly told us, "To be an excellent maths teacher you need both the subject matter and pedagogical content knowledge – and these certainly develop over time." This distinction between subject matter – knowledge about *maths* – and pedagogical content knowledge – knowledge about *teaching maths* – is important, although sometimes it might be something of a blurred boundary. In the second set of response pages after each *Espresso*, called *Positions*, you are invited to think about categorising types of knowledge like this and whether you find it helpful.

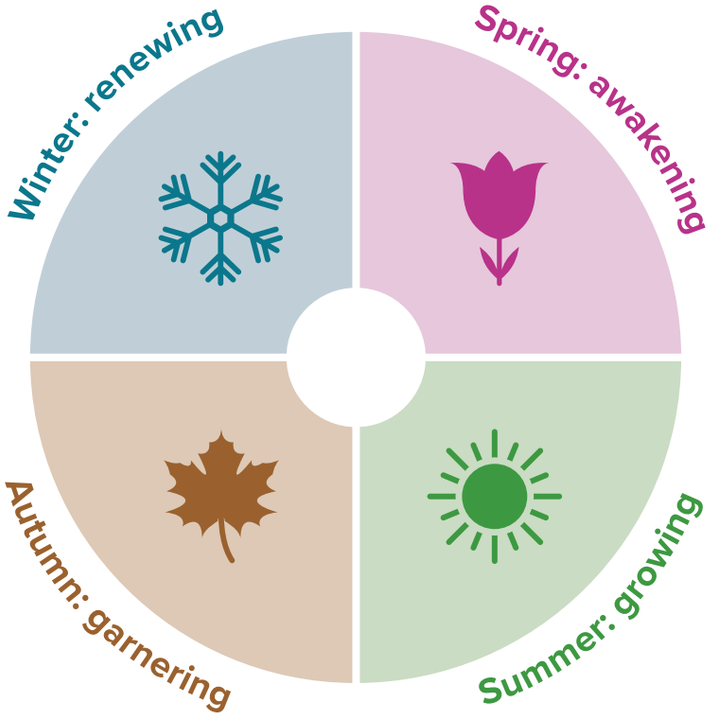
Jo Morgan in fact told us she had never felt like a "natural" teacher, but through hard work and experience she could become very good (if never brilliant). We think that kind of modesty and being hard on yourself is characteristic of maths teachers, too. Professor John Mason told us that he saw the process of becoming a maths teacher as "learning to balance care for learner(s) and care for mathematics." We think that "care for learners" here should also refer to caring for *yourself* as a learner. Reflecting on your practice as a teacher can be hard work and requires courage. Don't forget to make yourself a nice cuppa, perhaps with a biscuit of your choice, and remind yourself that you are doing a great job.

> "You are your best thing"
>
> **– Toni Morrison**

How to use this book

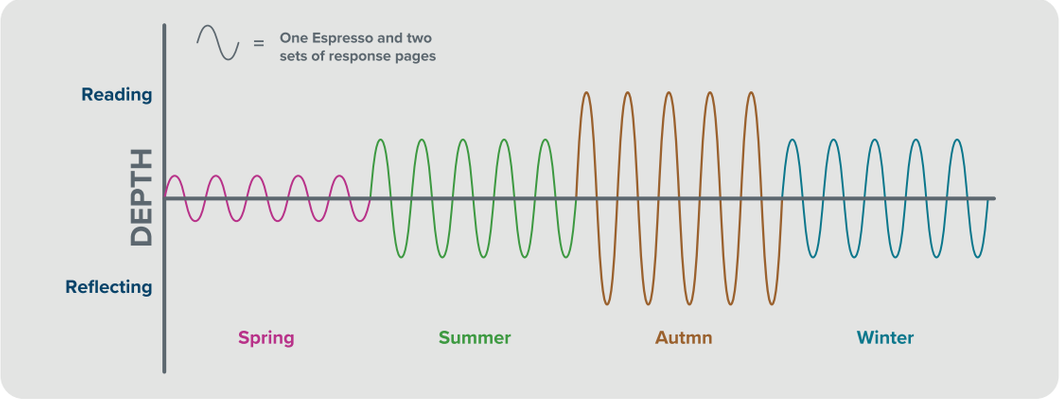
This book is not called a *Journeybook* for nothing. When writing it, we imagined what our perfect year of mathematics professional learning might look like – how the rhythm of the year might play out, with space and time for thinking, writing, talking, wondering, and doing.

By the end of the book (which may or may not take a year to work through) we hope that you will have been on a journey of sorts – perhaps even a kind of vacation – around the landscape of mathematics education research. As such, we have divided the book into four sections, named after four seasons, which correspond to different periods of growth in thinking about research:



The intention is for you to have the opportunity to do some activities repetitively – to practise. So, for example, in Spring, the set of questions in the *Expressions* section after each Espresso is the same. And then, you also have the opportunity to do some activities slightly differently – to move forwards. So, for example, in Summer, the set of questions in the *Expressions* section after each Espresso will switch to become slightly more in-depth and critical, and in Autumn even more so; Winter is a little gentler. We see this like a rhythmic cycle, (or a springy spring):

One way to visualise the Journeybook



So the order we have chosen for you to work through the Espressos in the book is not random, and the order of the types of reflective questions in each of the Spring, Summer, Autumn and Winter sections of the *Expressions* and *Positions* pages is not random either. The questions are generally gradually growing in depth, each one deliberately placed there as a landmark for you to explore on your journey.

But, like all good journeys, where you end up is your decision. If you choose to change the order in which you work through the Espressos within the seasonal sections, that is up to you. You can even, if you wish, work though the all the *Expressions* pages of the book before you work through the *Positions* pages, which are designed to be a little more in-depth. What we don't suggest is dipping in and out, either in terms of order or in terms of time – and this is because we know from research that good quality CPD of this type is usually sustained, intentional, reflective and rhythmic over time. It's also because we consider the connections between ideas in mathematics to be important, and the way that we have structured the Espressos in the book is so that you can make these connections for yourself as much as possible. For that reason, please also try to give each one time and space and don't skip any, even if it seems at first glance that it is not for you.

This book can be used in three different ways:

1. As an individual (perhaps with the option to talk about bits of it to others): for example, as part of self-directed CPD, to support work on a teacher development course, or to gain confidence as a Maths or Research Lead.

2. As a small group: for example, in whole-school or grouped CPD time, as a cluster across schools, or on a teacher development course (and each person should still have their own copy).

3. As the basis for mentoring/coaching: for example, between a mentor and mentee, to support a new member of staff, or between a Maths Lead and a member of staff looking to improve their skills in mathematics teaching.

However you decide to work through the book, the idea is very similar:

- Set aside a good chunk of time, ideally at regular intervals, for working through the book.
- Read the Espresso – once for "gist", and again for more detail.
- Feel free to draw, sketch, highlight or scribble on the Espresso itself.
- Work through the *Expressions* pages for the Espresso, directly in the book.
- Either in the same session, or another session if you would prefer, work through the *Positions* pages for the Espresso.
- Repeat!
- (And if you are working collaboratively, build in discussion time that allows people to share and challenge one another's ideas in an atmosphere of trust and safety.)
- At the end of each of the seasonal sections, there is a standalone extra page focused on one particular idea which you can work through in one session: *Maths teaching anxiety questionnaire*, *Beliefs about maths teaching questionnaire*, *Thinking about types of bias*, and *Thinking about types of evidence*.
- At the end of the book there is a signoff page that allows the book to be kept as a record of professional learning for the year.

Optional:

- If you wish, you can send us some of your completed pages, questions or feedback via **@CambridgeMaths** on Twitter or by email **admin@cambridgemaths.org**. Or write up your thoughts as a blog, as an article for one of the mathematics teacher association journals in your country, or as a magazine piece.
- If you can, arrange a workshop or a short talk where you can discuss and disseminate some of the ideas you have considered as part of working through the book.

Mathematical ideas … grow richer in meaning the more you play with them – each understanding brings a slightly different perspective.

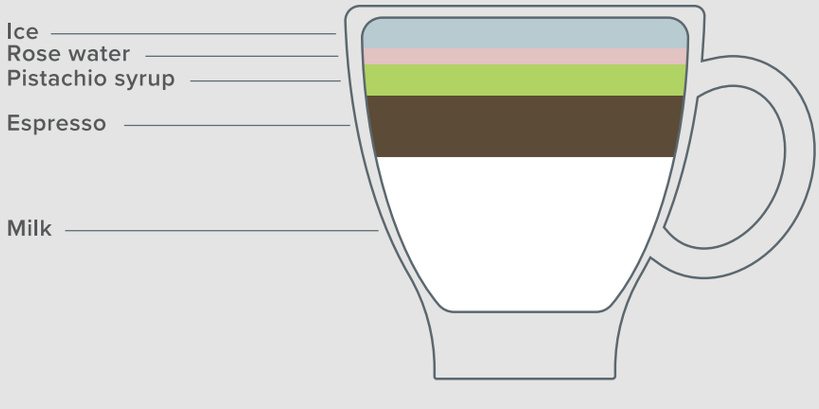
Francis Su, *Mathematics for Human Flourishing*, p. 38

Spring

An awakening of your views around educational research

- Thinking about what research is and who writes it
- Getting used to reading and rereading research summaries like *Espressos*
- Giving your views about research ideas
- Thinking about what you find useful and what you find less useful

Iced pistachio latte



5	:	2	:	1	:	1	:	0.5
Cold milk		Cold espresso		Pistachio syrup		Ice		Rose water

Ratios expressed in the glass not to scale