

Connecting Discrete Mathematics and Computer Science

Computer science majors taking a non-programming-based course like discrete mathematics might ask “why do I need to learn this?” Written with these students in mind, this text introduces the mathematical foundations of computer science by providing a comprehensive treatment of standard technical topics while simultaneously illustrating some of the broad-ranging applications of that material throughout the field. Chapters on core topics from discrete structures—like logic, proofs, number theory, counting, probability, graphs—are augmented with around 60 “Computer Science Connections” pages introducing their applications: for example, game trees (logic), triangulation of scenes in computer graphics (induction), the Enigma machine (counting), algorithmic bias (relations), differential privacy (probability), and paired kidney transplants (graphs). Pedagogical features include “Why You Might Care” sections, quick-reference “Chapter at a Glance” and Key Terms and Results summaries, problem-solving and writing tips, “Taking it Further” asides with more technical details, and around 1700 exercises, 435 worked examples, and 480 figures.

David Liben-Nowell is Professor of Computer Science at Carleton College, and earned degrees from Cornell, Cambridge, and MIT. His research focuses on computational social science, especially social networks. He teaches broadly, emphasizing introductory and theoretical computer science, and created and led a study-abroad program in the United Kingdom (History of Computing). He is on the ACM 202x curriculum subcommittee for mathematical foundations and is a member of the Liberal Arts CS (LACS) consortium. His awards and honors include NSF research funding, Kavli Fellow of the National Academy of Sciences, and Visiting By-Fellowship at Churchill College (Cambridge). He has also published about 30+ crossword puzzles in the *New York Times*, *Los Angeles Times*, *Wall Street Journal*, *Chronicle of Higher Education*, and other venues.

“An inspired approach to the introductory discrete math course, illuminating the aesthetic appeal of the subject together with the profound and inextricable links that connect it to the core ideas of computing.”

– Jon Kleinberg, Cornell University

“Unlike most discrete math texts, here the computer science content and connections are woven extensively throughout, with “forward pointers” that can excite students about numerous computer science areas they will encounter in their future studies. In addition, the book is written TO students, not FOR faculty. It will be a joy to teach with!”

– Valerie Barr, Mount Holyoke College

“Finally! I’ve spent years struggling to find a textbook that makes the topic of Discrete Structures relevant to computer science students. David Liben-Nowell has put forth a book that will make CS students invested in the material. He not only connects every topic to computer science, but does so in a clear and entertaining way.”

– Dan Arena, Vanderbilt University

“By foregrounding the connections between the fields, this outstanding textbook makes a compelling case for why computer science students should embrace the study of discrete mathematics. This is an approachable yet rigorous book, written with wit and verve, that I look forward to teaching from!”

– Raghuram Ramanujan, Davidson College

“David Liben-Nowell’s *Connecting Discrete Mathematics and Computer Science* provides students with a beautifully motivated, clearly written, and accessible exploration of the mathematical foundations of computer science. The “Computer Science Connections” sections provide compelling applications of the mathematical content, and the frequent “Taking it Further” notes provide extra richness that adds to the joy of the experience. This is a discrete math book that truly keeps the reader engaged!”

– Ran Libeskind-Hadas, Founding Chair of Integrated Sciences,
Claremont McKenna College

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David Liben-Nowell

Carleton College, Minnesota



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To MDSWM, with never-ending appreciation, and in loving memory of my grandfather, Jay Liben, who brought more joy, curiosity, and kvetching to this world than anyone else I know.

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That the proportion both of thanks and payment
Might have been mine! only I have left to say,
More is thy due than more than all can pay.

William Shakespeare (1564–1616)
The Scottish Play (c. 1606)

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David Liben-Nowell
 Northfield, MN
 August 2021

PS: I would be delighted to receive any comments or suggestions. Please don't hesitate to get in touch.

Credits

This book is a revised version of a book first published as *Discrete Mathematics for Computer Science* in 2017 (with a preliminary edition published in 2015) by John Wiley & Sons.

This book was typeset using \LaTeX , and I produced all but a few figures from scratch using TikZ. The other figures are reprinted with permission from their copyright holders:

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