1 Introduction

1.1 Family of Near-Zero-Index Materials/Metamaterials

The foundation of electromagnetic theory (1865) by James Clerk Maxwell and the pioneering experiment (1887) by Heinrich Hertz, exciting moments in physics, eventually led to the discovery of the electromagnetic wave. In the past one and a half centuries, the electromagnetic wave has played an essential role in wireless communication, target detection, remote sensing, medical engineering, and other fields, all of which substantially promote the prosperity of modern society. The past century has also witnessed an increasing interest in devices that provide better control of the transmission, scattering, and radiation of electromagnetic waves. Among the well-known examples are microwave waveguides, optical fibers, lenses and mirrors, and radio-frequency (RF) antennas and arrays, to name just a few. On the other hand, advances in material science and manufacturing crafts have opened exciting opportunities to conceive desired materials and structures at the level of microns and even nanometers. Assisted by elaborately designed materials, we are capable of sculpting wave–matter interactions in the deep subwavelength scale.

Generally, the macroscopic response of a material to the electromagnetic wave is depicted by its constitutive parameters: permittivity ε and permeability μ, presenting the electric and magnetic responses when the material is impinging on the electromagnetic wave. Another important material parameter relating to those two fundamental quantities is the optical refractive index $n = (\varepsilon\mu)^{1/2}$. Figure 1.1 illustrates the classification of various types of materials on the ε-μ plane, in which there are four quadrants with four different combinations of ε and μ. The free space has a permittivity ε_0 and a permeability μ_0, and the relative permittivity and permeability of a certain material are therefore defined as $\varepsilon_r = \varepsilon/\varepsilon_0$ and $\mu_r = \mu/\mu_0$, respectively. Most naturally occurring materials, such as dielectrics, feature a permeability and a permittivity larger than zero and hence are termed double positive materials, falling under the first quadrant of Fig. 1.1. As classified in the second quadrant, the materials with $\varepsilon < 0$ while $\mu > 0$ are designated as epsilon-negative materials. Many electric plasmonic materials behave in this manner below their plasma frequencies, such as the noble metals in the visible frequencies. The materials with $\varepsilon > 0$ while $\mu < 0$, categorized in the fourth quadrant, are called mu-negative materials. Some magnetic gyrotropic materials exhibit this characteristic in certain frequency regions. The double negative materials, lying in the third quadrant, also known as left-handed materials and negative refractive index (NRI) materials, were first proposed theoretically by V. G. Veselago in 1960s, but did not attract great interest for the next 30 years until their realization based on the emergent artificially structured materials (i.e., metamaterials [1–8]).

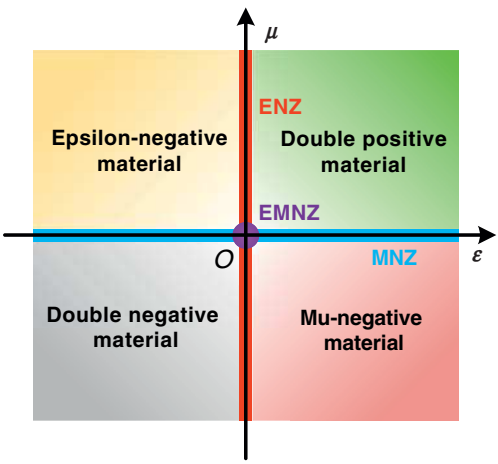

Figure 1.1 Classification of materials on the ε–μ plane.

Generally, the concept of metamaterial refers to a macroscopic composite of periodic or nonperiodic subwavelength resonant or nonresonant structures, which can be described by the effective constitutive parameters of a virtually homogenous material not existing in nature, such as double negative materials [1, 2]. The electromagnetic responses outside the materials are identical between the periodic metamaterial and the virtually homogenous material. The concept of metamaterials is widely studied and has performed well from microwave [3] to optical regions [4], and from general theory [5] to engineering applications [6]. Via suitable design of the unit cells and lattice configurations, metamaterials can be engineered to exhibit effective double positive, epsilon-negative, mu-negative, or double negative responses, promoting the development of metamaterials or meta-structures, as well as the emergence of various meta-devices [7]. Then, the three-dimensional bulky metamaterials evolve toward two-dimensional flatland structures (i.e., metasurfaces) for purposes of easy fabrication and advanced control of wave transmission and reflection [8].

The metamaterials falling under the transitional regions (shown as red and blue strips in Fig. 1.1) between two quadrants with refractive index near zero exhibit counterintuitive but interesting near-zero responses, such as optical nonlinearity enhancement, wave supercoupling, and directive emission [9, 10, 11]. Such a metamaterial is therefore called near-zero-index (NZI) metamaterial. In addition to the isotropic NZI metamaterials, anisotropic NZI metamaterials are also being proposed and investigated, whose constitutive parameter ε or μ is described by a tensor with one or several diagonal components close to zero [12]. Furthermore, the concept of NZI metamaterials can be readily extended to

other fields of physics, such as acoustics [13] and thermology [14], exhibiting excellent control ability for various types of waves. For example, the permittivity ε and permeability μ can be mapped onto the reciprocal of bulk modulus $1/\kappa$ and mass density ρ, respectively, in the acoustic wave equation. In this manner, it is possible to analyze acoustic NZI metamaterials in analogy with their electromagnetic counterparts. Concretely, depending on one or both constitutive parameters' approach to zero, NZI metamaterials can further be categorized as epsilon-near-zero ($\varepsilon \approx 0$, ENZ), mu-near-zero ($\mu \approx 0$, MNZ), and epsilon-and-mu-near-zero ($\varepsilon \approx 0$ and $\mu \approx 0$, EMNZ) metamaterials. The main difference among these three types of NZI metamaterials is the intrinsic impedance $Z = (\mu/\varepsilon)^{1/2}$, which is near zero for MNZ, near infinity for ENZ, and a regular complex number for EMNZ. The magnetic field is irrotational for ENZ, and the electric field is irrotational for MNZ. For EMNZ, both fields are irrotational.

In this Element, we focus on the ENZ metamaterials, performing a systematic review from the aspects of general concept, intriguing phenomena, and engineering applications. Corresponding to a particular zone in the constitutive parameter space, ENZ metamaterials are expected to yield different wave phenomena. We have known that the double positive and double negative materials or metamaterials are electromagnetically transparent, as the wave numbers in such media are real and the traveling wave state is supported. It can be understood that the single-negative materials/metamaterials are electromagnetically opaque due to the waves becoming evanescent in their bodies. By a simple inspection of the ε-μ plane in Fig. 1.1, one may raise a natural question: How would the electromagnetic waves propagate in ENZ metamaterials, which are situated in the critical zones? In fact, electromagnetic waves in the ENZ metamaterial exhibit a spatially static behavior, which is essentially different from that in other metamaterials. This question is addressed in Section 2 of this Element, and rich ENZ phenomena and functions are demonstrated and analyzed in the rest of the Element. Even though the natural ENZ materials (i.e., plasmonic materials) exist in the optical domain, nonetheless their intrinsic loss hinders the development of ENZ applications [15]. The idea of ENZ metamaterials using artificial structures to imitate ENZ behavior presents potential in practical applications. The key feature of this Element is presenting various engineering applications based on the concept of ENZ metamaterials.

1.2 History of Epsilon-Near-Zero (ENZ) Metamaterials

The pioneering work to analyze the wave dynamics in ENZ metamaterials may date back to 2004 [16]. In this work, Ziolkowski studied analytically and numerically the wave propagation and scattering in the ENZ medium and

revealed that the electromagnetic fields in the ENZ medium take on a static characteristic in space, yet remain temporally dynamic. The theory can also be adopted in other types of NZI medium with different impedance matching considerations. Arguably the first mind-bending phenomenon predicted in the ENZ material was that, as Silveirinha and Engheta theoretically demonstrated in 2006, the electromagnetic waves could be squeezed into a narrow two-dimensional ENZ channel and achieve a total transmission with near-zero reflection, an effect referred to as "supercoupling" [17]. Remarkably, the supercoupling effect depends neither on the length nor on the shape of the narrow ENZ channel, exhibiting substantial differences from the classic Fabry–Perot resonant transmission. In this work, the area of the ENZ channel should be infinitely small to achieve the total transmission property, which is also achieved by using $\mu \approx 0$ (i.e., EMNZ medium matching in this case). Subsequently in 2008, the supercoupling effect was experimentally validated using an extremely narrow waveguide operating at the cutoff frequency [18] and a metamaterial composed of planar complementary split-ring resonators inside a waveguide [19]. Both methods use a waveguide to imitate ENZ behavior but in different ways. In [19], a typical periodic metamaterial paradigm is used to achieve an ENZ effect. However, for [18], the waveguide operates at its cutoff frequency and can be treated as a new type of nonperiodic method to achieve an ENZ effect, which is further investigated in Section 2. In its early stage, interest in the ENZ metamaterials was strongly triggered by their potential to shape the radiation pattern and control the wave front. As discussed in Enoch's work (2002) [20], the stretched wavelength of electromagnetic waves in a low-index medium could provide the phase and magnitude uniformity of fields over an electrically large aperture and benefit the generation of a directive radiation for the broadside beam.

Subsequently, the roles that ENZ metamaterials play in wave–matter interactions were actively investigated and proved by experiment. In 2010, Ciattoni et al. theoretically analyzed the extremely nonlinear electrodynamics in an ENZ metamaterial that features a vanishing linear dielectric permittivity [21]. This work opened new horizons to boost the optical nonlinearity assisted by ENZ response, and a series of experiments was performed in nonlinear enhancement later. For example, Suchowski et al. in 2013 experimentally validated the phase-mismatch-free, four-wave-mixing process based on an ENZ metamaterial [22]. Besides enhancing materials' nonlinearity, ENZ metamaterials also have the potential to boost the extremely weak optical nonlocality. In Pollard's work (2009), the ENZ metamaterial is proved to be a desired platform for observing the spatial dispersion and additional wave, which do not exist in the local materials [23]. Another intriguing aspect of the wave–matter interaction in the

ENZ metamaterial is associated with the optical nonreciprocity. In 2013, Davoyan theoretically demonstrated that the magnetized ENZ metamaterial is a promising candidate to achieve one-way transmission, even with magnetically switched transparency and opacity [24]. ENZ metamaterials were also introduced into the regime of quantum optics, exhibiting functionalities in collective superradiance enhancement (2013) [25].

Another important timeline is for the development of the implementation schemes of ENZ metamaterials. As early as 1962, Rotman originally proposed to emulate plasmonic materials with the use of parallel-plate waveguides operating at the TE_{10} mode [26]. Specifically, the waveguide at the cutoff frequency of TE_{10} can mimic the behavior of ENZ material. Following this idea, in 2008 Edwards et al. fabricated waveguide ENZ metamaterials to verify the supercoupling effect in microwave frequency [18]. In the context of periodic artificial structures, ENZ metamaterials can also be equivalently realized via the zeroth-order mode of the left-handed transmission lines [27] and stacked alternative positive and negative dielectric layers [28]. In 2011, Huang et al. [29] showed that around the Dirac cone at the center of the Brillouin zone, the dielectric photonic crystal exhibits the desired zero-index property. Then it followed the work by Li et al. to realize the on-chip zero-index metamaterial (2015) [30] operating at the infrared frequency based on periodic structures. To tailor the macroscopic scattering properties of the ENZ metamaterial, Inigo et al. proposed the concept of photonic doping (2017) [31], employing dielectric impurities to control the effective permeability of the ENZ metamaterial. In 2019, Zhou et al. [32] introduced the substrate-integrated waveguide to realize the ENZ metamaterial and accommodated the concept of photonic doping in a planar architecture compatible with integrated circuits processes.

A handful of subfields in physics and engineering are rising, inspired by the ENZ metamaterials. An exciting subject is emergent optical metamaterial circuitries, which are discussed in detail in this Element. In 2005, Engheta et al. [33] demonstrated that deeply subwavelength non-plasmonic or plasmonic nanoparticles can serve as lump elements with effective capacitance or inductance, operating as nanocapacitors and nanoinductors for nanooptics. Subsequently in 2007, the concept of "metatronics" [34] was put forward to represent a class of metamaterial-inspired nanocircuits based on the scientific contribution in [33]. To connect those optical lump elements, the optical displacement-current conduit was proposed in 2009 by Alù et al. [35], who harnessed the ENZ metamaterial to efficiently confine and route the displacement current, analogous to the conduction current in DC circuits. Over the years, metatronics has been actively applied to design optical circuit modules; for example, optical filters with subwavelength scales [36]. In an important step

toward further progress, Li et al. in 2016 [37] explored the platform of the waveguide to implement the idea of metatronics, with much improved robustness and reduced optical losses from natural plasmonic materials [38, 39, 40].

1.3 Outline of the Element

This Element is dedicated to introducing the unusual wave phenomena arising from an extremely small optical index of refraction, and to shedding light on the underlying mechanisms, with the primary focus being on the basic concepts and fundamental wave physics. We reveal the exciting applications of ENZ metamaterials, which have profound impacts over a wide range of fields of science and technology. The sections are organized as follows. In Section 2, we demonstrate the extraordinary wave properties in ENZ metamaterials, analyzing the unique wave dynamics and the resulting effects. Section 3 introduces various realization methods of the ENZ metamaterials with periodic and non-periodic styles. The applications of ENZ metamaterials are discussed in Sections 4 and 5 from the perspectives of microwave engineering, optics, and quantum physics. We close in Section 6 by presenting an outlook on the development of ENZ metamaterials and discussing the key challenges to be addressed in future research.

2 Wave Properties in ENZ Metamaterials

2.1 Wave Dynamics at the ENZ Condition

In this section, we discuss the wave properties of ENZ metamaterials from the general electromagnetic theory and derive the counterintuitive characteristics of wave–matter interactions, including the basic wave dynamics, space and time decoupling, and wave supercoupling. Here, we start the discussion of the wave dynamics in the ENZ condition by firstly inspecting the basic equations in electromagnetics. Consider the case that a monochromatic plane wave propagates in an isotropic and homogenous medium and that the temporal period (i.e., frequency f) and the spatial period (i.e., wavelength λ) of the propagating wave are related via a simple expression:

$$f \cdot \lambda = c/\sqrt{\varepsilon_r \mu_r}, \tag{2.1}$$

where c is the speed of light in a vacuum, while ε_r and μ_r denote the relative permittivity and permeability of the medium. The right side of Eq. (2.1) is the phase velocity of light inside the medium. Consider that as ε_r approaches zero (i.e., ENZ condition), the phase velocity of light diverges to infinity. Under a given frequency f, it is also readily seen from Eq. (2.1) that the wavelength in

the medium should approach infinity as the relative permittivity ε_r vanishes to zero. Here the general conclusion is that when ε_r decreases to near zero, for different frequencies of the wave, the wavelength goes to infinity, the phase velocity goes to infinity, but the velocity of power flow goes to near zero, exhibiting static-like behavior inside the ENZ medium. Therefore, the diverging phase velocity and extremely stretched wavelength at the ENZ condition imply a suppressed variation of the electromagnetic fields in phase and amplitude over a large-scale space. Importantly, such a spatially static field configuration can be allowed at a nonzero frequency, which is a unique wave property in ENZ and other NZI materials.

To proceed, we check the ENZ limit ($\varepsilon_r \approx 0$) from the behavior of Maxwell's equations, in which two curl equations (i.e., Maxwell–Ampère law and Faraday's law) respectively read as follows:

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J} \tag{2.2}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{2.3}$$

where **E** and **H** are respectively the electric field and the magnetic field; $\mathbf{D} = \varepsilon_r \varepsilon_0 \mathbf{E}$ and $\mathbf{B} = \mu_r \mu_0 \mathbf{H}$ denote respectively the electric and magnetic flux densities. We assume that the area of interest is source-free (i.e., the current density $\mathbf{J} = 0$). At the ENZ condition ($\varepsilon_r \approx 0$), the electric flux density **D** vanishes, and therefore the curl of the magnetic field is zero, as seen from Eq. (2.2). Consequently, at the ENZ condition, the temporally varying magnetic field **H** is distributed in space the same way that the static magnetic field is. In particular, we consider a transverse-magnetic (TM) case where the magnetic field is polarized along one axis (e.g., the z-axis) and invariant along that axis, that is, $\partial_z \mathbf{H} = 0$. Substituting $H = H(x, y)\hat{z}$ into Eq. (2.2) and imposing the source-free and ENZ condition, we obtain $\nabla \times (H\ (x, y)\hat{z})$, which reduces to $H(x, y) \approx 0$. As seen, for this TM wave (oriented with respect to the z-axis) in the ENZ material, the magnetic field is a constant in any x-y cut plane. The spatially homogenous magnetic field distribution is a salient feature in two-dimensional (2D) ENZ materials or metamaterials, which is related to numerous unusual phenomena and functionalities. One of the most straightforward examples is the wave front manipulation. In the case of TM excitation, due to continuity of the tangential magnetic field, the wave front of the scattered or radiated magnetic field from an ENZ body should be conformal to its surface.

ENZ materials/metamaterials are also characterized by a diverging intrinsic impedance $\eta = \sqrt{\mu/\varepsilon}$, which results in the impedance mismatch with free space. As another general conclusion, when ε_r decreases to near zero, the intrinsic impedance of a wave inside the ENZ medium goes to infinity, presenting a value similar to that of a perfect magnetic conductor (PMC).

It is worth noting here that the infinite phase velocity at the ENZ condition does not contradict the causality principle in special relativity, because the phase velocity does not correspond to the velocity of either information or energy. What matters in the causality principle is the group velocity of the wave, which should be smaller than c. The group velocity in a dispersive homogeneous medium is defined and given by

$$v_g = \frac{\partial \omega}{\partial k} = \frac{2c}{2\sqrt{\mu_r \varepsilon_r} + \omega\sqrt{\frac{\mu_r}{\varepsilon_r}}\frac{\partial \varepsilon_r}{\partial \omega} + \omega\sqrt{\frac{\varepsilon_r}{\mu_r}}\frac{\partial \mu_r}{\partial \omega}}, \tag{2.4}$$

where $\omega = 2\pi f$ is the angular frequency and $k = 2\pi/\lambda$ is the wave number or propagation constant in the medium. In a lossless ENZ medium, the group velocity, evaluated by Eq. (2.4) after inserting $\mu_r = 1$, tends to zero as the slope of the permittivity as a function of angular frequency is finite. As discussed in [29], the NZI metamaterial with a single parameter (permittivity or permeability) approaching zero realized via a photonic crystal can feature a quadratic dispersion, $\delta\omega = 0 + O(\delta k^2)$, that is almost flat near $\mathbf{k} = 0$. As a case study, we consider a plasmonic material whose permittivity is described by the Drude-type dispersion: $\varepsilon_r(\omega) = 1 - \omega_p{}^2/(\omega^2 + i\omega\omega_c)$, where ω_p and ω_c are respectively the plasma frequency and the collision frequency. It is clear that in the lossless limit ($\omega_c \to 0$), the permittivity ε_r vanishes and the phase velocity of the wave diverges at plasma frequency ω_p, while the group velocity $v_g(\omega_p) = 2c\sqrt{\varepsilon_r(\omega_p)}/[\varepsilon_r(\omega_p) + 1]$ approaches zero. In fact, when the wave is excited in the ENZ medium, it requires a certain transitory time to build up the steady state of the spatially static field configuration, which ensures that the ENZ materials/metamaterials abide by the causality principle. On the other hand, for the practical system, the existing loss increases the group velocity inside the ENZ medium and also increases the transitory time needed to build up the spatially static field. For a regular ENZ medium with a certain amount of loss, the significantly reduced group velocity can be equivalently interpreted as the slowing down of light, which substantially increases the time scale of wave–matter interaction.

On the other hand, for the EMNZ metamaterials, the related intrinsic impedance $\eta = \sqrt{\mu/\varepsilon}$ is finite, since both the permeability μ and the permittivity ε

approach zero simultaneously, and so they can be interpreted as a kind of "matched" ENZ metamaterials [41]. The EMNZ medium is also known for its linear dispersion [29] and finite group velocity of light. By letting μ_r and ε_r approach zero while μ_r/ε_r and ε_r/μ_r are kept finite, it is easy to check in Eq. (2.4) that the group velocity v_g in a dispersive EMNZ medium is finite. In the EMNZ metamaterial realized via a photonic crystal with accidental degeneracy at $\mathbf{k} = 0$, a linear dispersion relationship of light can be obtained as $\delta\omega = v_g\ \delta k + o(\delta k)$ [29]. Recall the constitutive relationship $\mathbf{B} = \mu\mathbf{H}$ and that by imposing $\mu \approx 0$, Eq. (2.3) reduces to $\nabla \times \mathbf{E} \approx 0$, which is actually a high-frequency effective implementation of Kirchhoff's voltage law in a differential form. Evidently, in the EMNZ metamaterials, both the electric field and the magnetic field are spatially static, and furthermore, they no longer couple with each other. As systematically discussed in [42], a peculiar wave–matter interaction appears with "opening up" and "stretching" the space, as well as behaving electromagnetically as a single point even though the actual volume is electrically large. In this manner, the behavior of time-harmonic fields in an infinite EMNZ metamaterial can be fully analogous to that in electro- or magnetostatic fields. The electromagnetic wave with time convention exp(-$i\omega t$) suggests the exciting possibility of "DC optical circuits" (i.e., circuits operating at an optical frequency with a distributed wave characteristic), but the displacement current can be modeled as that in lumped circuits.

2.2 Decoupling of Spatial and Temporal Behavior of Waves

In this section, we pay specific attention to the property of decoupled temporal and spatial variation of electromagnetic fields in the ENZ medium and shed light on its underlying implications. Generally, as manifested in the wave equation,

$$\nabla^2 \begin{Bmatrix} \mathbf{E} \\ \mathbf{H} \end{Bmatrix} - \varepsilon\mu \frac{\partial^2}{\partial t^2} \begin{Bmatrix} \mathbf{E} \\ \mathbf{H} \end{Bmatrix} = 0, \tag{2.5}$$

the space-domain derivative operator ∇ and the time-domain derivative operator $\partial/\partial t$ are coexisting when describing a dynamic field behavior. The resulting effect is the coupled temporal and spatial variations of the electromagnetic fields. In the ENZ limit ($\varepsilon \approx 0$), however, the term associated with the time derivative ($\partial/\partial t$) in Eq. (2.2) vanishes to near zero, leading to the decoupling of temporal dynamic and spatial variation of the electromagnetic wave. This result arises from $\varepsilon \approx 0$, but it also can be effectively achieved when the time derivative is near zero, presenting static field behavior inside the ENZ medium. Such a result is also feasible for metamaterials with MNZ or EMNZ responses. To

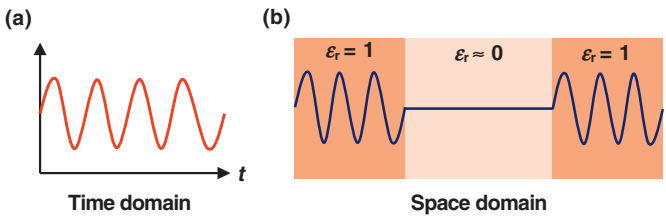

Figure 2.1 Decoupling of temporal and spatial variation of an electromagnetic field. Temporally oscillating field (a) with suppressed spatial variation in the ENZ region (b)

gain an intuitive understanding of this point, we consider a plane wave oscillating periodically in the time domain (illustrated in Fig. 2.1(a)) that transmits through an air–ENZ–air structure. A snapshot of the spatial distribution of the wave is schematically presented in Fig. 2.1(b). Although the wave remains temporally dynamic, it exhibits a spatially static behavior – uniform phase and amplitude distributions – in the ENZ region.

Due to the associated temporal and spatial properties of the electromagnetic wave, which are commonly observed in conventional materials and metamaterials, the operation frequencies of microwave and optical devices are usually related to their physical dimensions and detailed geometries. For example, the dipole antenna radiates efficiently when its physical length is about half the wavelength. The Fabry–Perot laser operates when the dimension of the cavity meets the resonance conditions. In this regard, ENZ metamaterials, which ratify the decoupled temporal and spatial wave properties, offer an exciting opportunity to conceive and realize geometry-irrelevant microwave/optical devices. As a representative example, let us consider geometry-irrelevant cavities made of ENZ metamaterials [43]. As conceptually illustrated in Fig. 2.2, three different cavities with arbitrary shapes are constructed with the 2D ENZ host as the background and a dielectric particle with cross-sectional area A_i. The ENZ hosts have different cross-sectional shapes but the same area, A_h. All the cavities are bounded by perfect electric conductor (PEC) walls. The resonance frequency of the cavity is formally defined as the eigenvalue of the source-free, time-harmonic wave equation subject to the PEC boundary condition ($\mathbf{n} \times \mathbf{E} = 0$). For this problem, we apply Faraday's law on the boundary of the ENZ host to yield [43]:

$$\omega = \frac{i}{\mu_0 A_\mathrm{h}} \frac{1}{H_h} \oint_{\partial \mathrm{A_i}} \mathbf{E} \cdot \mathrm{d}\mathbf{l}, \tag{2.6}$$