Random Matrix Methods for Machine Learning

This book presents a unified theory of random matrices for applications in machine learning, offering a large-dimensional data vision that exploits concentration and universality phenomena. This enables a precise understanding, and possible improvements, of the core mechanisms at play in real-world machine learning algorithms. The book opens with a thorough introduction to the theoretical basics of random matrices, which serves as a support to a wide scope of applications ranging from support vector machines, through semi-supervised learning, unsupervised spectral clustering, and graph methods, to neural networks and deep learning. For each application, the authors discuss small- versus large-dimensional intuitions of the problem, followed by a systematic random matrix analysis of the resulting performance and possible improvements. All concepts, applications, and variations are illustrated numerically on synthetic as well as real-world data, with MATLAB and Python code provided on the accompanying website.

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Preface

Numerous and large-dimensional data is now a default setting in modern machine learning (ML). Standard ML algorithms, starting with kernel methods such as support vector machines and graph-based methods like the PageRank algorithm, were however initially designed out of small-dimensional intuitions and tend to misbehave, if not completely collapse, when dealing with real-world large datasets. Random matrix theory has recently developed a broad spectrum of tools to help understand this new "curse of dimensionality," to help repair or completely recreate the suboptimal algorithms, and most importantly, to provide new intuitions to deal with modern data mining.

This book primarily aims to deliver these intuitions, by providing a digest of the recent theoretical and applied breakthroughs of random matrix theory into ML. Targeting a broad audience, spanning from undergraduate students interested in statistical learning to artificial intelligence engineers and researchers alike, the mathematical prerequisites to the book are minimal (basics of probability theory, linear algebra, and real and complex analyses are sufficient): As opposed to introductory books in the mathematical literature of random matrix theory and large-dimensional statistics, the theoretical focus here is restricted to the *essential* requirements to ML applications. These applications range from detection, statistical inference, and estimation to graph- and kernel-based supervised, semisupervised, and unsupervised classification, as well as neural networks: For these, a precise theoretical prediction of the algorithm performance (often *inaccessible* when not resorting to a random matrix analysis), large-dimensional insights, methods of improvement, along with a fundamental justification of the wide-scope applicability of the methods to real data, are provided.

Most methods, algorithms, and figures proposed in the book are coded in MATLAB and Python and made available to the readers (https://github.com/Zhenyu-LIAO/RMT4ML). The book also contains a series of exercises of two types: short exercises with corrections available online to familiarize the reader with the basic theoretical notions and tools in random matrix analysis, as well as long guided exercises to apply these tools to further concrete ML applications.