
Index

- Abelian, *see* group, Abelian
- action
 - on functions, 8
 - by homeomorphisms, 7
 - invariant subset, 8
 - invariant subspace, 325
 - left, 7
 - orbit, 8
 - orbit-stabilizer theorem, 8
 - right, 7
 - stabilizer, 8
 - by unitary operators, 7
- adapted measure, 27, 102, 309
- amenable
 - Følner condition, 163, 316
 - group, *see* group, amenable
 - Kesten's condition, 168
- anti-symmetric function, 122, 123
- aperiodic Markov chain, 79–81, 97
- Arzelà–Ascoli Theorem, 47, 270, 314
- Azuma's inequality, 180
- backward martingale convergence theorem, 65
- Banach space, 360
- Bass–Guivarc'h formula, 279
- Caccioppoli inequality, 333
- canonical path ensemble, 142
- capacity, 139, 141, 143
- Cayley graph, 26, 146
 - Cheeger constant, 162, 169
- central series, 15, 276
- centralizer, 249
- Cheeger constant, 163
- Choquet–Deny theorem, 216, 247, 248
- cocycle, 309
 - harmonic, 310, 313, 316, 318
- commutative, *see* group, Abelian
- conditional expectation, 51
 - tower property, 54, 200
- conductance, 118, 126, 127
 - effective conductance, 125, 126
- contraction, 164
- convolution, 9
- Coulhon–Saloff-Coste inequality, 169
- coupling, 371
- current, 124, 125, 131
- cutset, 138
- cycle, *see* path, cycle
- cylinder σ -algebra, 5
- detailed balance equation, 90
- Dirichlet energy, 121, 126, 139, 141, 143
- divergence, 120
- Doob's inequality, 61
- effective conductance, 125, 126
- eigenvalue
 - of a group automorphism, 285, 286, 288, 289
- EIT, 146
- electrical network, *see* network
- energy
 - Dirichlet, *see* Dirichlet energy
 - of a flow, 132
- entropy, 202
 - Avez entropy, *see* entropy, random walk
 - entropy
 - conditional entropy, 203
 - Kullback–Leibler divergence, 209
 - mutual information, 209

- random walk entropy, 206, 209, 228
- equilibrium measure, 140
- exponential growth, *see* group, exponential growth
- exponential intersection tails, 146
- exponential tail, 27, 102
- filtration, 54
 - canonical, 55
- finite index, 8, 101
- flow, 123, 132, 135
 - unit flow, 123, 131, 132, 135
- free group, 19, 216
- Fundamental theorem of finitely generated
 - Abelian groups, 12
- Følner sequence, 162, 316
- gambler's ruin, 59
- general linear group, 11, 285
- gradient, 120
- Green function, 128, 165, 184, 310
- Green's identity, 120, 167
- Grigorchuk group, 279
- Gromov's theorem, 146, 274, 302
- group
 - Abelian, 12, 281, 307
 - action, *see* action
 - affine, 25
 - amenable, 162, 219, 223, 311
 - center, 15
 - central series, 15, 276
 - centralizer, 249
 - characteristic subgroup, 282, 283
 - Choquet–Deny, 248
 - commutative, *see* group, Abelian
 - commutator, 4
 - commutator subgroup, 5
 - derived series, 17
 - dihedral, 25, 108
 - direct product, 24
 - exponential growth, 178, 264, 274, 275, 279
 - finite index, 8, 101
 - finitely generated, 5, 12, 20, 276
 - finitely presented, 22
 - free group, 19, 216
 - general linear group, 11, 285
 - Grigorchuk group, 279
 - growth, 274, 275
 - heat kernel bounds, 178
 - ICC, 252, 253, 256
 - indicable, 14, 302, 309, 329, 330
 - intermediate growth, 274, 275
 - isoperimetric dimension, 170
 - iterated commutator, 4
 - JNVN, 255, 256
 - lamplighter, 178, 220, 228
 - nilpotent, 15, 274–276, 279, 282, 288, 289, 291, 302
 - non-amenable, 170, 184
 - polynomial growth, 274, 275, 279, 302, 305, 307, 309, 329, 330, 332
 - recurrent, 141
 - semi-direct product, 24, 178, 284, 286
 - solvable, 17, 274, 279, 281, 291
 - special linear group, 12
 - sub-exponential growth, 220, 264, 280, 281, 291
 - transient, 141, 146
 - virtually nilpotent, 250
- group property, 14
 - virtual, 14, 142
- growth
 - exponential, 146, 178, 264, 274, 275, 279
 - of a group, 178, 274, 275, 279, 302, 305, 307, 309, 329, 330, 332
 - intermediate, 274, 275
 - polynomial, 146, 274, 275, 279, 302, 305, 307, 309, 329, 330, 332
 - sub-exponential, 220, 264, 280
- Hadamard's inequality, 337
- harmonic function, 29, 121, 131
 - bounded, 31, 65, 199, 201, 207, 209, 213, 252
 - cocycle, 310, 313, 316
 - into a Hilbert space, 309
 - Lipschitz, 32, 66, 317
 - Poisson formula, 201
 - polynomial growth, 33, 106
 - positive, 263, 264
- Hilbert space, 119, 303, 309–311, 359
 - Banach–Alaoglu theorem, 323
 - Hilbert–Schmidt norm, 321
 - Hilbert–Schmidt operator, 321, 324
 - invariant vectors, 319
 - orthogonal projection, 319, 359
 - orthonormal, 362
 - orthonormal basis, 362
 - separable, 320
 - von Neumann ergodic theorem, 319
 - weak* convergence, 322
- hitting measure, 101, 102, 105
- hitting time, 81
- Hoeffding's inequality, 180, 181

- Hölder's inequality, 10, 334
 horofunction, 32, 217
 index, *see* group, finite index
 indicable group, *see* group, indicable
 infinite conjugacy class (ICC), 252, 253, 256
 inner product, 358, *see also* Hilbert space
 integration by parts, 120, 167
 intermediate growth, 274, 275
 invariant
 event, 197
 σ -algebra, 197, 199, 201, 202, 209
 vector, 361
 irreducible Markov chain, *see* Markov chain, irreducible
 irreducible matrix, 29
 Jensen's inequality, 203
 Kesten's amenability criterion, 168
 Kirchhoff
 cycle law, 122, 135
 node law, 123
 Kleiner's theorem, 332, 336, 337
 Kolmogorov 0-1 law, 201
 Kolmogorov's extension theorem, 5
 lamplighter group, 178, 220, 228
 Laplacian, 121, 310
 linear functional, 360
 Liouville property, *see* random walk, Liouville
 Markov chain, 76
 aperiodic, 79–81, 97
 irreducible, 78, 80, 81, 97, 119
 lazy, 79
 recurrent, 85, 88, 95
 transient, 85, 88, 95
 Markov property, 76, 200
 martingale, 55, 124, 199
 bounded, 204
 martingale convergence theorem, 64, 124, 199, 200, 204
 maximal inequality, 61
 Milnor–Wolf theorem, 274, 279, 291
 moments, 27, 103, 105, 141, 145, 309
 Nash inequality, 173
 Nash–Williams criterion, 138
 network, 118
 Cheeger constant, 163
 connected, 119
 heat kernel bound, 175, 177
 isoperimetric dimension, 170
 isoperimetric inequality, 171, 172, 175
 Nash inequality, 173
 Sobolev inequality, 166, 171
 spectral radius, 164
 nilpotent, *see* group, nilpotent
 non-amenable group, 170, 184
 norm, 358, 360
 null recurrent, 89, 92, 95, 99
 operator norm, 164, 175
 optional stopping theorem (OST), 57, 124, 213
 orbit-stabilizer theorem, 8
 parabolic function, 199
 bounded, 199, 200, 207
 Poisson formula, 200
 parallelogram law, 360
 path
 cycle, 121, 135
 finite, 121
 infinite, 122
 in a network, 121
 reversal, 122, 130
 simple, 133, 138
 transient, 138
 ping-pong lemma, 19
 Pinsker's inequality, 211
 Poincaré inequality, 332, 333
 polarization identity, 360
 polynomial growth group, *see* group, polynomial growth
 positive recurrent, 89, 92, 95, 97
 predictable process, 63
 random walk
 adapted, 81
 Cheeger constant, 163
 diffusive, 317
 on a free group, 216
 on a graph, 78
 on a group, 28, 141, 330
 lazy, 78, 207
 Liouville, 201, 202, 209, 211, 213, 215, 219, 222, 257
 null-recurrent, 60
 on a network, 118
 range, 145
 recurrent, 56, 60, 85, 88, 95, 127, 141, 143, 144, 202, 222, 330
 speed, 215, 219
 transient, 85, 88, 95, 127, 129, 134, 139, 141, 143, 145, 184, 221
 on $\mathbb{Z}$, 56, 59, 62, 81, 86, 95, 139, 143, 181
 on $\mathbb{Z}^2$, 139, 145
 on $\mathbb{Z}^d$, 144, 216

- Rayleigh monotonicity, 127
- Reisz representation theorem, 360
- resistance, 125
 - effective resistance, 125, 126, 131
- reverse Poincaré inequality, 333
- Rosset's theorem, 281
- Sanov subgroup, 20
- semi-direct product, *see* group, semi-direct product
- SIT, 133, 137, 138, 146
- Sobolev inequality, 166, 171
- solvable, *see* group, solvable
- special linear group, 12
- spectral radius, 164
- stationary distribution, 89, 94, 95, 97
- stopping time, 55, 81
- strong Markov property, 83
- sub-exponential growth, *see* group, sub-exponential growth
- sub-martingale, 63
- summable intersection tails, *see* SIT
- super-martingale, 63
- symmetric
 - function, 122
 - matrix, 29
 - measure, 27, 102, 118, 309
 - subset, 4
- tail σ -algebra, 197, 199, 200, 206, 209
- Thompson's principle, 126, 127, 131, 140
- Tits alternative, 293
- Tonelli's theorem, 363
- total variation distance, 371
- transition matrix, 76–79, 81, 90, 95, 98, 118, 121, 310
- trivial σ -algebra, 201
- uniformly integrable, 56
- unitary
 - matrix, 303–305, 307, 309
 - operator, 303
- upcrossing lemma, 63
- Varopolous–Carne bound, 181, 182, 215
- voltage, 124, 125, 131
 - unit voltage, 124, 125, 131
- Young's inequality, 10