Introduction and overview

Cambridge Specialist Mathematics VCE Units 1&2 Second Edition provides a complete teaching and learning resource for the VCE Study Design **to be first implemented in 2023**. It has been written with understanding as its chief aim, and with ample practice offered through the worked examples and exercises. The work has been trialled in the classroom, and the approaches offered are based on classroom experience and the responses of teachers to earlier editions of this book and the requirements of the new Study Design.

The course is designed as preparation for Specialist Mathematics Units 3 and 4.

Specialist Mathematics Units 1 and 2 provide an introductory study of topics in proof, logic, sequences, **algorithms and pseudocode**, graph theory, algebra, functions, statistics, complex numbers, and vectors and their applications in a variety of practical and theoretical contexts. Techniques of proof are discussed in Chapter 6 and the concepts discussed there are employed in the following chapters and in Specialist Mathematics Units 3 and 4.

Chapter 1 provides an opportunity for students to revise and strengthen their algebra; and this is revisited in Chapter 3, where polynomial identities and partial fractions are introduced.

We have also written online appendices to support teachers and students to better develop their programming capabilities using **both the programming language Python and the inbuilt capabilities of students' CAS calculators**. Additional material on kinematics has also been placed in an online appendix.

Five extensive revision chapters are placed at key stages throughout the book. These provide technology-free multiple-choice and extended-response questions.

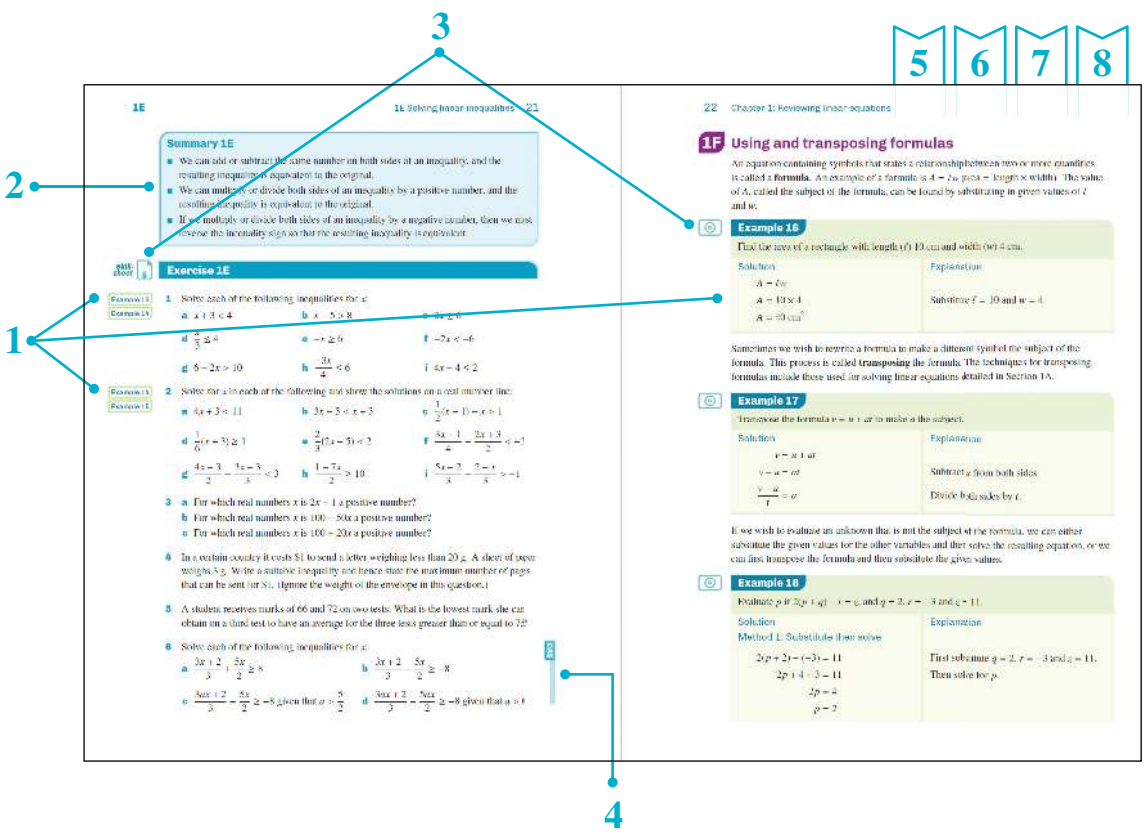
The first four revision chapters contain material suitable for **student investigations**, a feature of the new course. The Study Design suggests that '[a]n Investigation comprises one to two weeks of investigation into one or two practical or theoretical contexts or scenarios based on content from areas of study and application of key knowledge and key skills for the outcomes'. We have aimed to provide strong support for teachers in the development of these investigations.

The TI-Nspire calculator examples and instructions have been completed by Peter Flynn, and those for the Casio ClassPad by Mark Jelinek, and we thank them for their helpful contributions.

Overview of the print book

1. Graded step-by-step worked examples with precise explanations (and video versions) encourage independent learning, and are linked to exercise questions.
2. Section summaries provide important concepts in boxes for easy reference.
3. Additional linked resources in the Interactive Textbook are indicated by icons, such as skillsheets and video versions of examples.
4. Questions that suit the use of a CAS calculator to solve them are identified within exercises.
5. Chapter reviews contain a chapter summary and technology-free, multiple-choice, and extended-response questions.
6. Revision chapters provide comprehensive revision and preparation for assessment, including new practice Investigations.
7. The glossary includes page numbers of the main explanation of each term.
8. In addition to coverage within chapters, print and online appendices provide additional support for learning and applying algorithms and pseudocode, including the use of Python and TI-Nspire and Casio ClassPad for coding.

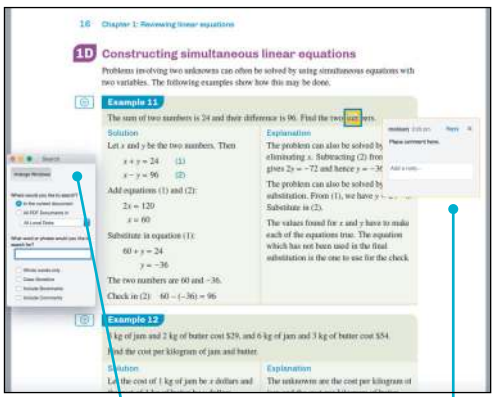
Numbers refer to descriptions above.



Overview of the downloadable PDF textbook

9 The convenience of a downloadable PDF textbook has been retained for times when users cannot go online.

10 PDF annotation and search features are enabled.



Overview of the Interactive Textbook

The **Interactive Textbook (ITB)** is an online HTML version of the print textbook powered by the HOTmaths platform, included with the print book or available as a separate purchase.

11 The material is formatted for on screen use with a convenient and easy-to-use navigation system and links to all resources.

12 **Workspaces** for all questions, which can be enabled or disabled by the teacher, allow students to enter working and answers online and to save them. Input is by typing, with the help of a symbol palette, handwriting and drawing on tablets, or by uploading images of writing or drawing done on paper.

13 **Self-assessment tools** enable students to check answers, mark their own work, and rate their confidence level in their work. This helps develop responsibility for learning and communicates progress and performance to the teacher. Student accounts can be linked to the learning management system used by the teacher in the Online Teaching Suite, so that teachers can review student self-assessment and provide feedback or adjust marks.

14 All worked examples have **video versions** to encourage independent learning.

15 **Worked solutions** are included and can be enabled or disabled in the student ITB accounts by the teacher.

16 An expanded and revised set of **Desmos interactives** and activities based on embedded graphics calculator and geometry tool windows demonstrate key concepts and enable students to visualise the mathematics.

17 The **Desmos graphics calculator, scientific calculator**, and **geometry tool** are also embedded for students to use for their own calculations and exploration.

18 **Revision of prior knowledge** is provided with links to diagnostic tests and Year 10 HOTmaths lessons.

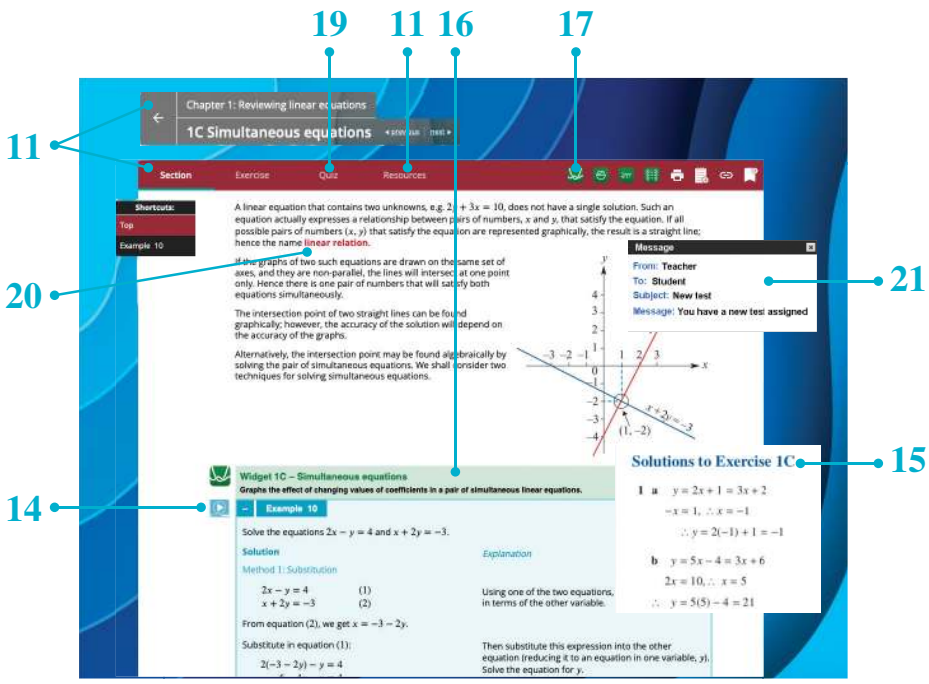
19 **Quick quizzes** containing automarked multiple-choice questions have been thoroughly expanded and revised, enabling students to check their understanding.

20 **Definitions** pop up for key terms in the text, and are also provided in a dictionary.

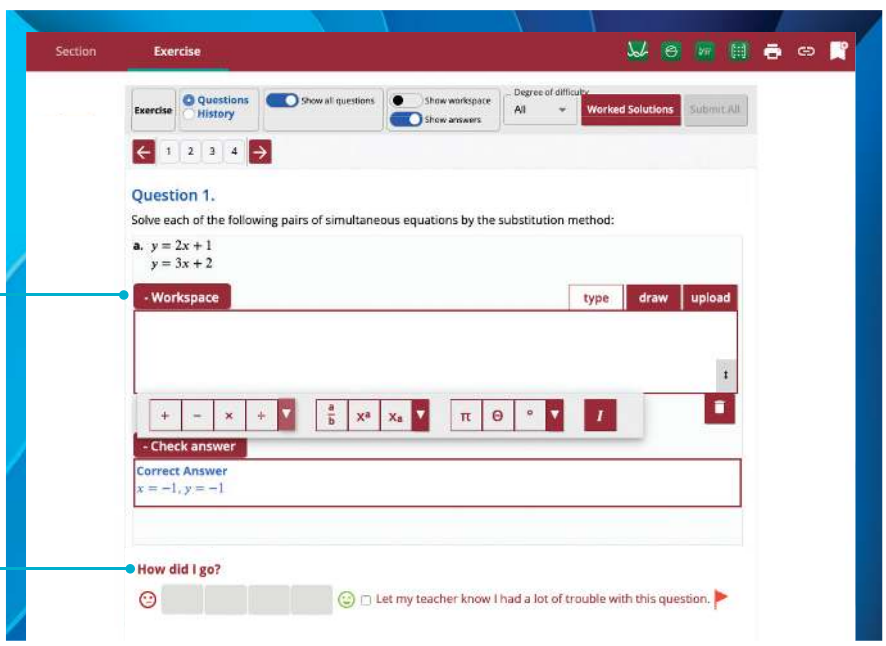
21 Messages from the teacher assign tasks and tests.

INTERACTIVE TEXTBOOK POWERED BY THE HOTmaths PLATFORM

A selection of features is shown. Numbers refer to the descriptions on pages xi–xii. HOTmaths platform features are updated regularly



WORKSPACES AND SELF-ASSESSMENT



Overview of the Online Teaching Suite powered by the HOTmaths platform

The Online Teaching Suite is automatically enabled with a teacher account and is integrated with the teacher's copy of the Interactive Textbook. All the teacher resources are in one place for easy access. The features include:

22 The HOTmaths learning management system with class and student analytics and reports, and communication tools.

23 Teacher's view of a student's working and self-assessment which enables them to modify the student's self-assessed marks, and respond where students flag that they had diffculty.

24 A HOTmaths-style test generator.

25 An expanded and revised suite of chapter tests, assignments and sample investigations.

26 Editable curriculum grids and teaching programs.

27 A brand-new **Exam Generator**, allowing the creation of customised printable and online trial exams (see below for more).

More about the Exam Generator

The Online Teaching Suite includes a comprehensive bank of VCAA exam questions, augmented by exam-style questions written by experts, to allow teachers to create custom trial exams.

Custom exams can model end-of-year exams, or target specific topics or types of questions that students may be having difficulty with.

Features include:

- Filtering by question-type, topic and degree of difficulty
- Searchable by key words
- Answers provided to teachers
- Worked solutions for all questions
- VCAA marking scheme
- Multiple-choice exams can be auto-marked if completed online, with filterable reports
- All custom exams can be printed and completed under exam-like conditions or used as revision.

Acknowledgements

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1 Reviewing algebra

Objectives

- To revise the **index laws**.
- To express numbers using **scientific notation**.
- To solve problems with **linear equations** and **simultaneous linear equations**.
- To use **substitution** and **transposition** with formulas.
- To add and multiply algebraic fractions.
- To solve **literal equations**.
- To solve **simultaneous literal equations**.

Algebra is the language of mathematics. Algebra helps us to state ideas more simply. It also enables us to make general statements about mathematics, and to solve problems that would be difficult to solve otherwise.

We know by basic arithmetic that $9 \times 7 + 2 \times 7 = 11 \times 7$. We could replace the number 7 in this statement by any other number we like, and so we could write down infinitely many such statements. These can all be captured by the algebraic statement $9x + 2x = 11x$, for any number x. Thus algebra enables us to write down general statements.

Formulas enable mathematical ideas to be stated clearly and concisely. An example is the well-known formula for compound interest. Suppose that an initial amount P is invested at an interest rate R, with interest compounded annually. Then the amount, A_n, that the investment is worth after n years is given by $A_n = P(1 + R)^n$.

In this chapter we review some of the techniques which you have met in previous years. Algebra plays a central role in Specialist Mathematics at Years 11 and 12. It is important that you become fluent with the techniques introduced in this chapter and in Chapter 4.

1A Indices

This section revises algebra involving indices.

Review of index laws

For all non-zero real numbers a and b and all integers m and n:

- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $(ab)^n = a^n b^n$
- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
- $a^{-n} = \frac{1}{a^n}$
- $\frac{1}{a^{-n}} = a^n$
- $a^0 = 1$

Rational indices

If a is a positive real number and n is a natural number, then $a^{\frac{1}{n}}$ is defined to be the nth root of a. That is, $a^{\frac{1}{n}}$ is the positive number whose nth power is a. For example: $9^{\frac{1}{2}} = \sqrt{9} = 3$.

If n is odd, then we can define $a^{\frac{1}{n}}$ when a is negative. If a is negative and n is odd, define $a^{\frac{1}{n}}$ to be the number whose nth power is a. For example: $(-8)^{\frac{1}{3}} = -2$.

In both cases we can write:

$$a^{\frac{1}{n}} = \sqrt[n]{a} \quad \text{with} \quad \left(a^{\frac{1}{n}}\right)^n = a$$

In general, the expression a^x can be defined for rational indices, i.e. when $x = \frac{m}{n}$, where m and n are integers, by defining

$$a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m$$

To employ this definition, we will always first write the fractional power in simplest form.

Note: The index laws hold for rational indices m and n whenever both sides of the equation are defined (for example, if a and b are positive real numbers).

Example 1

Simplify each of the following:

a $x^2 \times x^3$ **b** $\frac{x^4}{x^2}$ **c** $x^{\frac{1}{2}} \div x^{\frac{4}{5}}$ **d** $(x^3)^{\frac{1}{2}}$

Solution	Explanation
a $x^2 \times x^3 = x^{2+3} = x^5$	$a^m \times a^n = a^{m+n}$
b $\frac{x^4}{x^2} = x^{4-2} = x^2$	$\frac{a^m}{a^n} = a^{m-n}$
c $x^{\frac{1}{2}} \div x^{\frac{4}{5}} = x^{\frac{1}{2}-\frac{4}{5}} = x^{-\frac{3}{10}}$	$\frac{a^m}{a^n} = a^{m-n}$
d $(x^3)^{\frac{1}{2}} = x^{\frac{3}{2}}$	$(a^m)^n = a^{mn}$

Example 2

Evaluate:

a $125^{\frac{2}{3}}$ **b** $\left(\frac{1000}{27}\right)^{\frac{2}{3}}$

Solution

a $125^{\frac{2}{3}} = \left(125^{\frac{1}{3}}\right)^2 = 5^2 = 25$

b $\left(\frac{1000}{27}\right)^{\frac{2}{3}} = \left(\left(\frac{1000}{27}\right)^{\frac{1}{3}}\right)^2 = \left(\frac{10}{3}\right)^2 = \frac{100}{9}$

Explanation

$125^{\frac{1}{3}} = \sqrt[3]{125} = 5$

$\left(\frac{1000}{27}\right)^{\frac{1}{3}} = \sqrt[3]{\frac{1000}{27}} = \frac{10}{3}$

Example 3

Simplify $\dfrac{\sqrt[4]{x^2y^3}}{x^{\frac{1}{2}}y^{\frac{2}{3}}}$.

Solution

$$\frac{\sqrt[4]{x^2y^3}}{x^{\frac{1}{2}}y^{\frac{2}{3}}} = \frac{(x^2y^3)^{\frac{1}{4}}}{x^{\frac{1}{2}}y^{\frac{2}{3}}} = \frac{x^{\frac{2}{4}}y^{\frac{3}{4}}}{x^{\frac{1}{2}}y^{\frac{2}{3}}}$$
$$= x^{\frac{2}{4}-\frac{1}{2}}y^{\frac{3}{4}-\frac{2}{3}}$$
$$= x^0y^{\frac{1}{12}}$$
$$= y^{\frac{1}{12}}$$

Explanation

$(ab)^n = a^nb^n$

$\frac{a^m}{a^n} = a^{m-n}$

$a^0 = 1$

Summary 1A

- Index laws
  - $a^m \times a^n = a^{m+n}$
  - $a^m \div a^n = a^{m-n}$
  - $(a^m)^n = a^{mn}$
  - $(ab)^n = a^nb^n$
  - $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
  - $a^{-n} = \frac{1}{a^n}$
  - $\frac{1}{a^{-n}} = a^n$
  - $a^0 = 1$
- Rational indices
  - $a^{\frac{1}{n}} = \sqrt[n]{a}$
  - $a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m$

Exercise 1A

Example 1

1 Simplify each of the following using the appropriate index laws:

a $x^3 \times x^4$ **b** $a^5 \times a^{-3}$ **c** $x^2 \times x^{-1} \times x^2$ **d** $\frac{y^3}{y^7}$

e $\frac{x^8}{x^{-4}}$ **f** $\frac{p^{-5}}{p^2}$ **g** $a^{\frac{1}{2}} \div a^{\frac{2}{3}}$ **h** $(a^{-2})^4$

i $(y^{-2})^{-7}$ **j** $(x^5)^3$ **k** $(a^{-20})^{\frac{3}{5}}$ **l** $\left(x^{-\frac{1}{2}}\right)^{-4}$

m $(n^{10})^{\frac{1}{5}}$ **n** $2x^{\frac{1}{2}} \times 4x^3$ **o** $(a^2)^{\frac{5}{2}} \times a^{-4}$ **p** $\dfrac{1}{x^{-4}}$

q $\left(2n^{-\frac{2}{5}}\right)^5 \div (4^3n^4)$ **r** $x^3 \times 2x^{\frac{1}{2}} \times -4x^{-\frac{3}{2}}$

s $(ab^3)^2 \times a^{-2}b^{-4} \times \dfrac{1}{a^2b^{-3}}$ **t** $(2^2p^{-3} \times 4^3p^5 \div (6p^{-3}))^0$

Example 2

2 Evaluate each of the following:

a $25^{\frac{1}{2}}$ **b** $64^{\frac{1}{3}}$ **c** $\left(\dfrac{16}{9}\right)^{\frac{1}{2}}$ **d** $16^{-\frac{1}{2}}$

e $\left(\dfrac{49}{36}\right)^{-\frac{1}{2}}$ **f** $27^{\frac{1}{3}}$ **g** $144^{\frac{1}{2}}$ **h** $64^{\frac{2}{3}}$

i $9^{\frac{3}{2}}$ **j** $\left(\dfrac{81}{16}\right)^{\frac{1}{4}}$ **k** $\left(\dfrac{23}{5}\right)^0$ **l** $128^{\frac{3}{7}}$

3 Use your calculator to evaluate each of the following, correct to two decimal places:

a 4.35^2 **b** 2.4^5 **c** $\sqrt{34.6921}$ **d** $(0.02)^{-3}$ **e** $\sqrt[3]{0.729}$

f $\sqrt[4]{2.3045}$ **g** $(345.64)^{-\frac{1}{3}}$ **h** $(4.568)^{\frac{2}{5}}$ **i** $\dfrac{1}{(0.064)^{-\frac{1}{3}}}$

4 Simplify each of the following, giving your answer with positive index:

a $\dfrac{a^2b^3}{a^{-2}b^{-4}}$ **b** $\dfrac{2a^2(2b)^3}{(2a)^{-2}b^{-4}}$ **c** $\dfrac{a^{-2}b^{-3}}{a^{-2}b^{-4}}$

d $\dfrac{a^2b^3}{a^{-2}b^{-4}} \times \dfrac{ab}{a^{-1}b^{-1}}$ **e** $\dfrac{(2a)^2 \times 8b^3}{16a^{-2}b^{-4}}$ **f** $\dfrac{2a^2b^3}{8a^{-2}b^{-4}} \div \dfrac{16ab}{(2a)^{-1}b^{-1}}$

5 Write $\dfrac{2^n \times 8^n}{2^{2n} \times 16}$ in the form 2^{an+b}.

6 Write $2^{-x} \times 3^{-x} \times 6^{2x} \times 3^{2x} \times 2^{2x}$ as a power of 6.

7 Simplify each of the following:

a $2^{\frac{1}{3}} \times 2^{\frac{1}{6}} \times 2^{-\frac{2}{3}}$ **b** $a^{\frac{1}{4}} \times a^{\frac{2}{5}} \times a^{-\frac{1}{10}}$ **c** $2^{\frac{2}{3}} \times 2^{\frac{5}{6}} \times 2^{-\frac{2}{3}}$

d $\left(2^{\frac{1}{3}}\right)^2 \times \left(2^{\frac{1}{2}}\right)^5$ **e** $\left(2^{\frac{1}{3}}\right)^2 \times 2^{\frac{1}{3}} \times 2^{-\frac{2}{5}}$

Example 3

8 Simplify each of the following:

a $\sqrt[3]{a^3b^2} \div \sqrt[3]{a^2b^{-1}}$ **b** $\sqrt{a^3b^2} \times \sqrt{a^2b^{-1}}$ **c** $\sqrt[5]{a^3b^2} \times \sqrt[5]{a^2b^{-1}}$

d $\sqrt{a^{-4}b^2} \times \sqrt{a^3b^{-1}}$ **e** $\sqrt{a^3b^2c^{-3}} \times \sqrt{a^2b^{-1}c^{-5}}$ **f** $\sqrt[5]{a^3b^2} \div \sqrt[5]{a^2b^{-1}}$

g $\dfrac{\sqrt{a^3b^2}}{a^2b^{-1}c^{-5}} \times \dfrac{\sqrt{a^{-4}b^2}}{a^3b^{-1}} \times \sqrt{a^3b^{-1}}$