Historians constantly wrestle with uncertainty, never more so than when attempting quantification; yet, the field has given little attention to the nature of uncertainty and strategies for managing it. This volume proposes a powerful new approach to uncertainty in ancient history, drawing on techniques widely used in the social and natural sciences. It shows how probability-based techniques used to manage uncertainty about the future or the present can be applied to uncertainty about the past. A substantial introduction explains the use of probability to represent uncertainty. The chapters that follow showcase how the technique can offer leverage on a wide range of problems in ancient history, from the incidence of expropriation in the Classical Greek world to the money supply of the Roman Empire.

MYLES LAVAN is Reader in Ancient History at the University of St Andrews. He is the author of Slaves to Rome: Paradigms of Empire in Roman Culture (Cambridge, 2013) and the co-editor of Roman and Local Citizenship in the Long Second Century CE (2021) and Cosmopolitanism and Empire: Universal Rulers, Local Elites, and Cultural Integration in the Ancient Near East and Mediterranean (2016).

DANIEL JEW is Senior Lecturer in the Department of History and Director of Studies at the College of Alice & Peter Tan, within the National University of Singapore. He is the co-editor of M. I. Finley: An Ancient Historian and His Impact (Cambridge, 2016) and the author of several papers on agriculture, craft, women and slaves in the ancient Greek economy. He is writing a monograph on carrying capacity in Athens.

BART DANON is Assistant Professor in Ancient History at Groningen University. He works on the social and economic history of the Roman Empire, with a focus on inequality, the political economy, urbanisation and slavery.
The Uncertain Past

Probability in Ancient History

Edited by

MYLES LAVAN
University of St Andrews

DANIEL JEW
National University of Singapore

BART DANON
Groningen University
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Contributors

**Gilles Bransbourg** is the Executive Director of the American Numismatic Society in New York and a Research Associate at the Institute for the Study of the Ancient World at New York University.

**Bart Danon** is Assistant Professor in Ancient History at Groningen University.

**Mohamed Elhouderi** is a computer engineer working for Sopra Steria Benelux.

**J. W. Hanson** is Associate Lecturer in the Department of Classics at the University of Reading.

**Daniel Jew** is Senior Lecturer at the Department of History and Director of Studies at the College of Alice & Peter Tan, within the National University of Singapore.

**Paul V. Kelly** is a retired consulting actuary, currently studying at the London School of Economics.

**Myles Lavan** is Reader in Ancient History at the University of St Andrews.

**Emily Mackil** is Professor of History at the University of California, Berkeley.

**Nicolas Solonakis** is a postdoctoral researcher associated with the European Research Council project ‘Patrimonium’ (Bordeaux Montaigne University).

**Anicet Touré** is an engineer working for Tractebel Engineering – Engie Group.
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