

QUANTUM MEASUREMENT

This book adopts a novel, physics-first approach to quantum measurement, using physical experiments as the basis to describe the underlying mathematical formalism. Topics covered include weak measurements, quantum measurement reversal, quantum trajectories, and the stochastic path-integral formalism. The theory of quantum measurement is also covered in detail, including discussion of how it can be tested and demonstrated in a laboratory: how to build quantum-limited amplifiers, fundamental noise limits imposed on measurement by quantum mechanics, and the design of superconducting circuits. This text is an excellent introduction for students with a basic understanding of quantum mechanics wanting to learn more about measurement theory, and the inclusion of a wide selection of end-of-chapter exercises makes this book ideal for emerging courses on the topic. Key chapters introducing the foundations of quantum computing and the history of measurement theory are equally accessible to a broader, less specialized audience.

ANDREW N. JORDAN received his B.S. from Texas A&M and his Ph.D. in physics from University of California–Santa Barbara. He serves on the faculty at the University of Rochester and Chapman University where he is co-Director of the Institute for Quantum Studies. Andrew has received the NSF CAREER award and a Simons Fellowship in theoretical physics for his work.

IRFAN A. SIDDIQI received his A.B. from Harvard University and his Ph.D. in applied physics from Yale University. He is a member of the Physics and Electrical Engineering faculty at the University of California–Berkeley and faculty scientist at Lawrence Berkeley National Laboratory. Irfan is a fellow of the American Physical Society and has received numerous awards including the George E. Valley, Jr. Prize, the John F. Keithley Award, and the University of California–Berkeley Distinguished Teaching Award.

QUANTUM MEASUREMENT

Theory and Practice

ANDREW N. JORDAN

Chapman University and University of Rochester

IRFAN A. SIDDIQI

University of California–Berkeley



Cambridge University Press & Assessment
978-1-009-10006-9 — Quantum Measurement
Andrew N. Jordan , Irfan A. Siddiqi
Frontmatter
[More Information](#)



Shaftesbury Road, Cambridge CB2 8EA, United Kingdom
One Liberty Plaza, 20th Floor, New York, NY 10006, USA
477 Williamstown Road, Port Melbourne, VIC 3207, Australia
314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India
103 Penang Road, #05–06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of Cambridge University Press & Assessment,
a department of the University of Cambridge.

We share the University's mission to contribute to society through the pursuit of
education, learning and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781009100069

DOI: 10.1017/9781009103909

© Cambridge University Press & Assessment 2024

This publication is in copyright. Subject to statutory exception and to the provisions
of relevant collective licensing agreements, no reproduction of any part may take
place without the written permission of Cambridge University Press & Assessment.

First published 2024

A catalogue record for this publication is available from the British Library

A Cataloging-in-Publication data record for this book is available from the Library of Congress

ISBN 978-1-009-10006-9 Hardback

Cambridge University Press & Assessment has no responsibility for the persistence
or accuracy of URLs for external or third-party internet websites referred to in this
publication and does not guarantee that any content on such websites is, or will remain,
accurate or appropriate.

Cambridge University Press & Assessment
978-1-009-10006-9 — Quantum Measurement
Andrew N. Jordan , Irfan A. Siddiqi
Frontmatter
[More Information](#)

**To our wives, children, and families. Marian, Thomas, Juliana, Catherine,
James, Terence, Sylvia, Peter and John. Lutfur-Rehman, Qudsia, Ambreen,
Talha, and Mariam.**

For all you have done to support us in our work

Contents

<i>Preface</i>	<i>page</i> xi
<i>Acknowledgments</i>	xiv
<i>List of Symbols</i>	xv
1 Introduction to Quantum Physics and Measurement	1
1.1 Prologue	1
1.2 The Era of the Founders: 1920s–1950s	2
1.3 The Era of Bell: 1960–1970s	4
1.4 Classic Experiments: 1970–1980s	10
1.5 The Quantum Information Era: 1990s–Present	19
1.6 Generalized Measurements	21
1.7 What You Will Learn in This Book	26
2 Projective Measurement	30
2.1 The Stern–Gerlach Experiment	30
2.2 Measurements on Multiple Systems	35
2.3 Mathematics of Projective Measurement	37
2.4 Continuous Variables	42
2.5 Discussion of the Cardinal Properties of Projective Measurement	42
3 Generalized Measurement	46
3.1 Measuring the Polarization of a Single Photon	46
3.2 Measuring Polarization with Position	49
3.3 Polarization State Update	50
3.4 Mathematics of Generalized Measurement	53
3.5 von Neumann’s Model: An Example with a Qubit and Free Particle Meter	54
	vii

viii	<i>Contents</i>	
	3.6 Generalization to Mixed States	57
	3.7 Quantum Bayesian Point of View	60
4	Weak Measurement	64
	4.1 The Limit of a Very Weak Stern–Gerlach Magnet	64
	4.2 Information–Disturbance Trade-off	66
	4.3 Weak Value	71
	4.4 Weak Value Amplification	75
	4.5 Generalized Eigenvalues for Any Measurement Type	81
5	Continuous Measurement: Diffusive Case	90
	5.1 Measuring the Location of an Electron on a Double Quantum Dot with a Quantum Point Contact	91
	5.2 Measuring the State of a Superconducting Quantum Circuit with Electromagnetic Radiation	97
	5.3 Stochastic Schrödinger and Master Equations	104
	5.4 Continuous Measurement	108
	5.5 Stochastic Path Integral	120
	5.6 Diffusive Measurement with Continuous Variables	124
6	Continuous Measurement: Quantum Jump Case	129
	6.1 Blinking Atoms and Their Emitted Photons	130
	6.2 Quantum Jumps in Superconducting Qubits	133
	6.3 Continuous Nature of Quantum Jumps	135
	6.4 Modified Stochastic Quantum Equations of Motion	138
7	Linear Detectors	153
	7.1 Quantum Noise and Measurement Limits	155
	7.2 Linear Response Theory	156
	7.3 Quantum Limited Pointer States	161
8	Quantum Amplification	169
	8.1 Quantum-Noise-Limited Operation	169
	8.2 Superconducting Josephson Tunnel Junction Circuits	171
	8.3 Degenerate Parametric Amplifier	173
	8.4 Standing-Wave Amplifier Circuits	176
	8.5 Traveling-Wave Amplifier Circuits	179
9	Measurement-Related Phenomena and Applications	186
	9.1 Measurement Reversal	186
	9.2 Most Likely Path	189

<i>Contents</i>	ix
9.3 Joint Measurement of Noncommuting Observables	199
9.4 Entanglement by Measurement: Direct and Indirect	205
10 Feedback and Control	218
10.1 General Theory of Feedback	218
10.2 Continuous Quantum State Analog Stabilization	220
10.3 Canonical Phase Measurement	223
10.4 Continuous Error Correction	228
11 Epilogue: What Does It All Mean?	233
11.1 What Quantum Mechanics Is and Is Not	233
11.2 Charting the Quantum Technological Frontier	234
11.3 What We Have Learned and the Road Ahead	235
11.4 Beyond Quantum	238
<i>Appendix A</i> Review of Classical Probability Theory	239
<i>Appendix B</i> Mixed Quantum States	243
<i>References</i>	249
<i>Index</i>	261

Preface

The subject of quantum measurement is now over a hundred years old, and yet we are still discovering new facets of it. Remarkably, despite the chaotic historical origins of quantum theory, its pillars have withstood the scientific test of time. While the mathematical formulations of quantum measurements have been demonstrated to be correct, they do not tell you “how it works.” Over time, we have achieved a deeper and deeper understanding of this physics, with an increasing set of phenomena associated with it. Indeed, in past decades with the introduction of ever-improving quantum technology, there has been a quiet series of groundbreaking experiments that have pushed our best theories of quantum measurement in new directions. These discoveries have allowed physicists to experimentally probe the most fundamental aspects of quantum measurement and, for the first time, to “look under the hood” to understand the dynamics and statistics of the quantum measurement process. These findings help to elucidate many of the puzzling features of quantum physics: What is wavefunction collapse? How can quantum-limited measurements be made? How can quantum systems be monitored continuously? What is the connection between physical reality and information? Can we describe the measurement quantum mechanically? And so on.

However, in our experience, most physicists’ understanding of measurement is stuck somewhere between 1920 and 1980. Even the most mathematically advanced treatments of quantum physics have a very crude understanding of how quantum measurements work, either in theory or in practice. The three cardinal properties of textbook measurements, of being projective, irreversible, and instantaneous, are idealizations and are usually only approximations to experimental reality. How to generalize and adapt the basic concepts of measurement to more realistic situations was unexplained until fairly recently. This text will guide the reader through the theory of generalized quantum measurements, the role of measurement strength and time, as well as mathematical formulations of continuous measurement. We

will also explain how these new types of quantum measurements are designed and carried out in different fields of physics.

A number of books have been published that discuss quantum measurement and related fields: see Mensky (1993); Carmichael (1993); Braginsky and Khalili (1995); Percival (1998); Breuer and Petruccione (2002); Gardiner and Zoller (2004); Haroche and Raimond (2006); Barchielli and Gregoratti (2009); Wiseman and Milburn (2010); Jacobs (2014). We mention in particular the classic text by Braginsky and Khalili, focusing mainly on interferometric light techniques, and of great practicality to the Laser Interferometer Gravitational-wave Observatory (LIGO), as well as the book of Wiseman and Milburn, focusing on the mathematical development of the theory of quantum measurement, particularly for atomic and optical systems, complemented by the book of Carmichael, all pioneers in this area. The book of Jacobs, as well as that of Breuer and Petruccione, address a wide variety of topics on this and related subjects in great detail. We also mention the helpful introductions of Jacobs and Steck (2006) and Brun (2002).

Sometimes books tend toward the extremes of the mathematical or the encyclopedic and usually are sparse on the experimental aspects of the subject. In addition, there are many philosophical texts on the subject of quantum measurement, of varying degrees of quality. Popular books (and movies!) on the subject (that we will not cite here) can be cringeworthy products written by novices who know just enough to be dangerous, or by scientists that have an axe to grind – usually related to issues of philosophy. Additionally, there have been several new experiments pushing the boundaries of our understanding and technical ability, as well as new theoretical formalism and insights. Most importantly, many of the subjects discussed here do not appear in any book we know of. This situation has convinced the authors that we are at a time when a new book on the subject, aimed at a wide audience, is needed. Our approach here is to write an accessible scientific book on the subject that is grounded in experimental findings.

It is our intention that much of this book can profitably be read by advanced undergraduates with some knowledge of quantum physics. The main audience is graduate students, postdocs, and other scientists wanting an accessible introduction and limited survey of the field. The literature is too vast for a complete treatment, so we focus on what we feel are the essential aspects of the field and give ample references to the original literature for further learning. Our goal is to concatenate the theory canon into a streamlined tool chest for quantum “mechanics” and to give experimentalists the techniques and methods needed for carrying out a wide variety of quantum measurements.

Preface

xiii

Information is the most spiritual thing in the physical world.

- Andrew N. Jordan

Quantum mechanics is the science of describing a vector that lives in an imaginary world, can only be reconstructed through our eyes in certain directions, and even those glances have already shaken it from its slumber.

- Irfan A. Siddiqi



Laguna Beach, CA May 13, 2022

Andrew N. Jordan (left side of both photos) and Irfan A. Siddiqi (right side of both photos), celebrating at the Montage Hotel in Laguna Beach, CA, after finishing this book.

Acknowledgments

A complete list of thanks to all those supporting this book is impossible, so we highlight just a few people who were most closely involved.

Thanks go to Philippe Lewalle, who helped make some of the figures for the book; to Sasha Korokov, for many discussions about quantum mechanics; and to the late Markus Büttiker, for his mentorship. The excellent cover illustration of an artistic rendering of a superconducting quantum circuit was made by graphic artist Peter Jordan. We thank our colleagues for many discussions over the years that have helped refine our understanding of the field and spurred on the discoveries described in this book, including Joe Eberly, Justin Dressel, and Howard Wiseman. It is a pleasure and privilege to acknowledge our students and postdocs over our careers, who taught us as much as we taught them.

We owe a huge thanks to the government agencies and foundations that have supported our research reflected in this book. In particular, we thank the US Army Research Office, especially T. R. Govindan and Mike Metcalfe. Other supporting institutions are the National Science Foundation, the Department of Energy, the US Air Force Office of Science, the John Templeton Foundation, the Simons Foundation, and our affiliations: Chapman University, the University of California–Berkeley, and the University of Rochester.

We thank Justin Dressel, Michel Devoret, Alexandre Blais, Nico Roch and Kevin O’Brien for providing critical comments, and Daniel Esteve, for urging us to write this book. Thanks also go to Sarah Carranza, for her help in the final stages of preparing the book for publication. Finally, we thank the Cambridge University Press team for their support in the production and publication of this text.

List of Symbols

α	Coherent state amplitude
\mathbf{A}	Vector notation
χ	Dispersive frequency shift
Δ	Tunneling rate or Detuning
δ	Dirac or Kronecker delta function
ϵ	Energy asymmetry
η	Efficiency
Γ	Rate
\hbar	Reduced Planck constant
κ	Generalized eigenvalue or Cavity escape rate
$\langle \dots \rangle$	Expectation value
λ	Eigenvalue or Wavelength or Coupling constant
μ	Magnetic moment
ω	Frequency
\overline{A}	Matrix notation
Φ	Magnetic flux
ϕ	Phase
Φ_0	Magnetic flux quantum
Π	Projection operator
ψ	Quantum state
τ_0	Correlation time
τ_m	Characteristic measurement time
Θ	Temperature
ξ	Langevin random variable
$*$	Complex conjugate
\dagger	Hermitian conjugate
A	Amplitude

B	Magnetic field
C	Capacitance or Bhattacharyya coefficient or Concurrence
c	Speed of light in a vacuum
D	Displacement operator or Bhattacharyya distance
d	Degree of decoherence
dW	Wiener increment
E	POVM element or Electric field
e	Electron charge or Euler’s number
E_C	Charging energy
E_F	Fermi energy
E_J	Josephson energy
F	Force
G	Conductance or Gain
h	Planck constant
I	Electrical current
L	Inductance
n	Number of Cooper pairs
P	Probability
p	Momentum, or Probability density
r	Measurement result
R_q	Resistance quantum
S	Noise spectral density or Scattering matrix
T	Duration of time
V	Voltage or Potential
W	Wiener random variable
x, y, z	Bloch coordinates
Y	Spherical harmonic
Z	Impedance or Partition function
\bar{A}	Average of A
\mathcal{H}	Stochastic Hamiltonian
\mathcal{L}	Lindbladian or Lagrangian density
\mathcal{M}	Murity
\mathcal{N}	Wigner–Smith time delay matrix
Q	Accumulated charge
\mathcal{R}	Signal-to-noise ratio
\mathcal{S}	Stochastic action
\mathcal{T}	Time-ordering operator or Transmission
$\hat{\Omega}$	Kraus (or measurement) operator
$\hat{\rho}$	Density operator
$\hat{\sigma}$	Unnormalized density operator or Pauli operator

List of Symbols

xvii

$\hat{\Theta}$	Time-reversal operator
$\hat{a}, \hat{b}, \hat{c}$	Bosonic annihilation operators
\hat{H}	Hamiltonian operator
\hat{L}	Lindblad operator
\hat{U}	Unitary operator
\hat{X}, \hat{P}	Quadrature operators
\hat{O}	Operator or Observable