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Introduction to Quantum Physics and Measurement

1.1 Prologue

Quantum measurement sometimes has a bad reputation. No less an authority than J. S. Bell once wrote an article entitled "Against 'measurement'" (Bell, 1990). Philosophers of physics are not usually fond of it either, usually only referring to it indirectly as the "measurement problem." Practitioners of the physics of quantum measurement are sometimes derisively labeled as "instrumentalists." Why then should we devote a whole book to the subject? The simplest answer is that the word *measurement* is essentially a shorthand for how we interact with the world. It is, strictly speaking, the only way we can gain information about the world, and only from the data we collect can we begin to formulate theories about the world around us.

In the context of quantum mechanics, as students and researchers of the field, we spend so much time dwelling on the mathematics of the subject – quantum states, observables, Hilbert spaces, symmetries, equations of motion, and so on – we sometimes forget that the results of experiments and every lived experience we have of the world involves none of those things directly. It only involves measurement. Without measurement, none of those mathematical objects would have any scientific meaning. It is this tendency to sweep measurement under the rug, as well as our role as an observer, that has, in our view, held back the further elucidation of quantum physics. Thus, while some physicists think of wavefunction collapse as the "ugly scar" on an otherwise beautiful theory (Gottfried, 1989), it is only by taking its scientific role seriously that we can make further progress in the understanding of quantum physics.

While both philosophy and mathematics will arise in this book, it is our goal to have a "physics first" approach to this subject. In the founders' era of quantum mechanics, direct experimental evidence of many of the predictions of quantum mechanics was lacking because the necessary technology did not exist. This resulted in most of the "experiments" from that time being *Gedankenexperiments*, that is, thought experiments about what might happen. In the subsequent decades, the situation has changed radically. Scientists and engineers have developed many new instruments and physical systems to be able to explore and test new phenomena and extreme limits of the theory to unprecedented levels. There are many new deep and exciting theoretical predictions and concurrent experiments in this subject that have come up in the past decades. In the chapters that follow, we will guide you though some of them. By putting experimental phenomena first, followed by a mathematical description, we will gradually gain an intuitive understanding of how quantum measurement works as an operational science.

Philosophical matters are nevertheless important. The past half-century or more has led to a sprouting up of numerous "interpretations" of quantum mechanics. A clear understanding about the various mathematical objects and physical predictions in the theory can lead to new understanding. These interpretations are a metanarrative, riding over the theory and attempting to give further meaning to the theoretical content as well as draw out possible philosophical implications. These interpretations are too often an ex post facto forcing of physical phenomena into contorted mental constructs. This brings us to our maxim regarding interpretations of quantum mechanics: the best interpretation is a fruitful interpretation. It is only by leading one to new insights and discoveries that an interpretation becomes more than a stumbling block. In the epilogue that concludes this book, we will give an outlook on the metaphysical understanding of the field and how interpretation can guide us in physical discoveries.

1.2 The Era of the Founders: 1920s–1950s

Quantum theory developed in the 1910s with the introduction of the Bohr model of the atom, and quickly entered its modern form in the 1920s with the complementarity principle of Bohr, the uncertainty principle of Heisenberg, and the wave equation of Schrödinger. Measurement entered quantum mechanics via a postulate which stated that interrogating a system results in a final state that is a single stationary state, or eigenstate, of the measurement apparatus. A system may at any time prior to this be in a superposition of such eigenstates, and measurement collapses the wavefunction into a well-defined outcome whose probability does not evolve in time. One notes that it is therefore not possible to directly access the full wavefunction of a system in a single measurement, and measurement necessarily causes disturbance. What then does the wavefunction actually represent?

While we are more comfortable now with the idea that all accessible information can be packed into a complex wavefunction that can be extracted in certain allowed combinations using mathematical operators, the interpretation put forth by Max Born in around 1926 in a series of papers that linked the modulus squared of the wavefunction with the probability density to find the system it described was a tremendous step forward in linking abstract quantum quantities with measurement outcomes (Born, 1926). Born noted that the coefficient associated with a given eigenfunction, when the wavefunction is expanded in the basis of the measurement apparatus, represents the probability amplitude that such an outcome would be observed. Born received the 1954 Nobel Prize in Physics "for his fundamental research in quantum mechanics, especially for his statistical interpretation of the wavefunction."

The measurement postulate, as originally conceived, is perfectly compatible with all past and future experimental results tested thus far. Nonetheless it is ad hoc and leaves much to the imagination with respect to the details of the measurement process itself. While a system evolves according to the Schrödinger equation, measurement was put forth as an "instantaneous" collapse of the wavefunction – a process that inherently assumes that information extraction is too fast to access, and is at odds with the fact that any physical apparatus has a characteristic measurement time. Moreover, there are many experimental situations that are not naturally suited to this canonical description. For example, the spontaneous emission decay of an atom ultimately results in the occupation of the ground state whether the atom was always in the ground state from the start or arrived there after a photon emission. More subtly, the measurement rate from the point of view of an external observer is not on equal footing for these two outcomes. For a relaxation event, a rapid quantum jump is the signature of the process, whereas if no such jumps are detected, we must wait much longer than the average decay lifetime to safely conclude the atom was originally in the ground state.

As such, since the early foundations of quantum mechanics, there was extensive discussion of the role of measurement, which was quite important in the thinking behind the Heisenberg uncertainty principle (Heisenberg, 1985). One of the most influential and systematic early thinkers in this area was Hungarian-American John von Neumann. In his classic text *Mathematical Foundations of Quantum Mechanics* (Von Neumann, 1955), originally published in 1932, von Neumann advanced an important line of research taking seriously measurement as a physical process. He recognized that an instantaneous measurement of, for example, energy would run afoul of the time-energy uncertainty principle, and that therefore it is not possible to carry out an arbitrarily precise measurement in a very short amount of time. This led him to develop a dynamical model of the measurement process, whereby the information about the system of interest can be extracted by coupling it to an auxiliary degree of freedom, or meter.

The meter, or probe, variable could be treated within a fully quantum mechanical framework, and then itself be detected using a recording device via the projection postulate. This procedure will be referred to as von Neumann's measurement model. It is an important advance because the combined system/meter/recording device is given a dynamical description, with its own Hamiltonian and equations of motion. This model can be seen as the precursor to later mathematical models of quantum dynamics, including decoherence theory, weak measurement, and quantum trajectories that will be addressed in later chapters of this text.

Nevertheless, the concept of instantaneous wavefunction collapse sparked much debate, including the celebrated paradox put forward by Einstein, Podolsky, and Rosen (EPR). A pair of entangled objects cannot, by definition, be described by concatenating independent descriptions of each constituent piece. Correspondingly, measurement outcomes of each element of the pair must be correlated, seemingly implying that if entanglement exists between physically separated objects, then quantum mechanics is capable of instantaneous, nonlocal influences. This led to the EPR paradox (Einstein et al., 1935), which raised the apparent incompatibility of this notion with locality since the information needed to generate the correlations associated with quantum mechanics would need to be exchanged faster than the speed of light if the two parts of an entangled pair are spacelike separated. This led EPR to conclude that the quantum description of reality as given by a wavefunction is not complete. Without further thought experiments that could prove the existence of quantum entanglement while simultaneously establishing the inability of any classical theory to predict these unique measurement outcomes, and without the experimental tools to access the EPR regime, this dilemma of, as Einstein put it, "Spooky Action at a Distance" lay dormant and unexplored until the arrival of John S. Bell (Fig. 1.1).

1.3 The Era of Bell: 1960–1970s

Alternative Formalisms to Cope with Reality

During the Second World War, science was mostly on hold. Notable exceptions are, of course, the Manhattan Project and development of the atomic bomb, harnessing nuclear physics for the war effort. Following that period, research in quantum mechanics was focused mostly on relativistic generalizations and on the growing field of particle physics. The 1960s was the golden age of field theory, giving rise to the systematic categorization of the many different particles that were being discovered, leading to the so-called standard model of particle physics. The existence of quarks proposed by Murray Gell-Mann and George Zweig in 1964 was verified with the discovery of all the quark species, the last being the top quark discovered

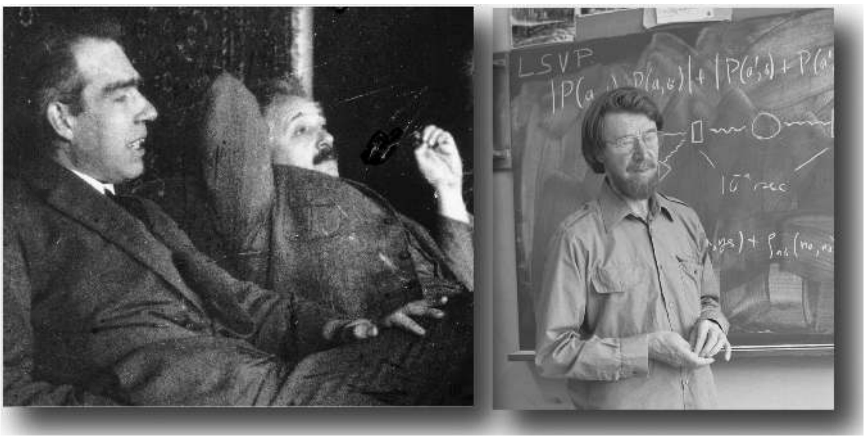

Figure 1.1 From left to right: Niels Bohr and Einstein debating the nature of reality. John Bell at the board, decorated with his famous experiment. Reprinted with permission from the Niels Bohr Library & Archives, American Institute of Physics, and CERN.

in 1995 at Fermilab. The final missing particle, the Higgs boson, proposed independently in three papers in 1964 by Brout, Englert, Higgs, Guralnik, Hagen, and Kibble, was finally discovered at the CERN particle collider in 2013.

While there were important contributions to quantum mechanics in this period (we mention, for example, Feynman's path integral formulation in 1948 (Feynman, 1948) and the Aharonov–Bohm effect in 1959 (Aharonov and Bohm, 1959)), there was not much activity on what we may call the "fundamentals" of quantum mechanics. However, some scientists were unhappy with the philosophical implications of the dominant quantum interpretation, the "Copenhagen interpretation," most systematically described by the Danish physicist Niels Bohr. They were following in the footsteps of Albert Einstein, who, although a co-founder of the field, was deeply disturbed by the intrinsic randomness of the theory, and the lack of a clear "realist" view of what quantum theory describes. In classical physics, we encounter the notion of randomness only as a lack of complete knowledge – for example, whether a flipped coin lands "heads" or "tails" is simply our practical inability to quickly integrate Newton's equations from the given initial condition. The mathematical description of classical probability theory is given in Appendix A. However, in quantum theory, to the best of our knowledge, probability theory is inescapable. Identically prepared initial conditions can give rise to different final outcomes. Thus the concept of preexisting properties before measurement is generally rejected in quantum theory. This concept of a system's property, like position, momentum, energy, and so on, existing without any reference to measurement or observers is what we mean by "classical realism."

One of the more influential scientists of this era was David Bohm (1917–1992), who advanced another view of quantum mechanics. Bohm, together with Ahanonov, reformulated the EPR argument (originally made for continuous position and momentum variables) in terms of measurements of components of spin-1/2 particles. They pointed out that currently it was practicable to test it experimentally only in the study of the polarization properties of correlated photons (Bohm, 2012; Bohm and Aharonov, 1957). Bohm had interpreted quantum physics in 1952 as describing an underlying real particle degree of freedom that was "guided" by an unobservable wave, sometimes called a "pilot" wave (Bohm, 1952a; Bohm, 1952b), called by Bohm a "hidden variable," drawing from the earlier "hidden parameter" of John von Neumann (1955). This view of quantum mechanics is mathematically the same as that of Madelung's hydrodynamic formulation of quantum mechanics (Madelung, 1926; Madelung, 1927) and philosophically similar to the early views of de Broglie (1927). It avoided the "no-go" hidden parameter theorem of von Neumann by also giving the detector an active role to play in the particle dynamics. The Bohmian interpretation had the feature of giving a clear mental picture of the particle degree of freedom, but suffered a number of drawbacks. When applied to more than one particle, such as in the Einstein, Podolsky, Rosen thought experiment (Einstein et al., 1935) (questioning the completeness of quantum mechanics), the pilot wave must act instantaneously across space and time in order to "guide" the particle to the correct detector result. That feature, together with an infinite number of variant theories possible, and difficulties in making a relativistically invariant version (which was already well established within standard quantum mechanics), led most physicists to discount it. However, the fact that such a reinterpretation of quantum theory was possible in the first place was an advance in foundations of quantum mechanics and motivation for further thought.

Bell Correlations, Testing Differences with Hidden Variable Theories

The issue lying in the background is whether quantum mechanics, like statistical mechanics, is an emergent theory describing a yet more fundamental physics of unobserved degrees of freedom, the abovementioned hidden variables. Further, if these hidden variables, exist, the thinking goes, perhaps they will restore our view of objective reality – where physics describes properties that exist before we measure them. The existence of the Bohm reinterpretation led John S. Bell (1928–1990) to think more broadly about the problem and to see what properties *any* theory that involved this type of hidden variable would have. In a groundbreaking paper published in 1964 in an obscure journal *Physics*, which only published four volumes between 1964 and 1968 (Bell, 1964), Bell published his inequality. The result of the inequality puts up a scientific conflict between a class of "hidden variable" theories and quantum mechanics. The scientific nature of this inequality is quite important – it is not an interpretation of quantum theory, but an experimental test that these hidden variable theories must satisfy. The basic setup is that two particles, called S_1 and S_2, are spatially separated such that no communication between them is possible that is faster than the speed of light, and then each is measured to collect data on its properties by two different measuring devices. Bell put in the ingredients that a naive scientific realist would want in a theory for this simple case:

- That the properties of each particle, called A and B, to be measured have definite, preexisting values, determined by the hidden variable.
- That statistical outcomes of measurements are simply the result of averaging over the hidden variables (which are produced randomly at the creation of the two-particle system with some unspecified distribution), as is the case in statistical mechanics.
- The "vital" assumption is that the result B for particle 2 does not depend on the detector setting for particle 1, nor result A for particle 1 on the detector setting of particle 2. Bell quotes Einstein: "But on one supposition we should, in my opinion, absolutely hold fast: the real factual situation of the system S_2 is independent of what is done with the system S_1, which is spatially separated from the former" (Schilpp, 1949). The spatial separation should be sufficient that no causal influence can be transmitted faster than the speed of light, in accordance to the principles of relativity theory.

Some modern commentators also argue there is an implicit assumption of freedom of choice; the experimenters operating the measuring devices of particles 1 and 2 are free to change their settings at will (there is no superdeterminism). There are also more exotic hidden assumptions like no retrocausality.

In his six-page paper, Bell goes on to show that this class of "local realistic" hidden variable theories (local because of the third assumption, realistic because of the first) make definite predictions about the statistical result of measurements made with different settings of the two measurement devices: a correlation function of the measured data has certain bounds on its value, no matter what the distribution of hidden variables. Consequently, any experiment that violates the Bell bound on local realistic hidden variable theories will rule such theories out. Furthermore, quantum physics can violate that bound. It should be noted that not all hidden variable theories fit into this category. For example, the hidden variable theory of Bohm contradicts assumption 3, allowing nonlocal influences.

After Bell's paper was published, the next obvious question was to test it: which was right, the above hidden variable theories, or quantum mechanics? An important step was the introduction of a modified inequality in 1969 by John Clauser, Michael Horne, Abner Shimony, and R. A. Holt, which was the test usually implemented in the experiments (Clauser et al., 1969). There was a series of optical experiments testing the preceding inequality, focusing on the polarization degree of freedom of two entangled photons. In particular, we mention the 1972 experiment of Freedman and Clauser (1972), which shows Bell's inequality was indeed violated. Interestingly, the 1973 experiment by Holt and Pipken using atomic mercury to produce two-photon events at Harvard University (Holt, 1973) showed that Bell's inequality was *satisfied*, casting doubt on quantum mechanics! Clauser repeated the experiment (Clauser, 1976) and found, together with the independent experiment of Fry and Thompson (1976), that Bell's inequality was indeed violated in this type of system.

This created a series of Bell-type experiments that continue to the present day. Of great interest is the possibility of ruling out "loopholes" in the experiments, whereby some experimental imperfections or design flaw may not truly rule out local realistic hidden variables. We mention in particular the influential 1981 and 1982 experiments of Alain Aspect (Fig. 1.3) and coworkers Grangier, Roger, and Dalibard (Aspect et al., 1981; Aspect et al., 1982), the latter of which was the first to implement fast random switching of the polarization analyzers, faster than the light travel time between the measured systems. This ruled out the possibility of the setting of one polarizer being able to causally influence the outcome of the other (nonlocal) photon. Physicists also started violating Bell's inequality over longer and longer distances, such as the work of Nicholas Gisin's group showing violation using Swiss Telecom lines between two villages in the Geneva vicinity, separated by 18 km (Salart et al., 2008), testing the speed of the "spooky action at a distance." Recent notable experiments along these lines are a series of the experiments in 2015 designed to rule out two loopholes (also closing the "fair-sampling" loophole) in the same experiment (Hensen et al., 2015; Giustina et al., 2015; Shalm et al., 2015). An event-ready Bell experiment was subsequently made (Rosenfield, 2017). These experiments are the most aggressive tests of local hidden variable theories to date and all showed that Bell's inequality remained violated. Consequently, we are stuck with quantum mechanics unless we want to break one or more of the assumptions of Bell's theorem. For these experiments the 2022 Nobel Prize in Physics was awarded jointly to Alain Aspect, John F. Clauser, and Anton Zeilinger "for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science."

Decoherence Became Fashionable with Zurek and Kraus

It is clear that dissatisfaction with ideas of quantum measurement is behind much of the work of Bohm, Bell, and others. This idea crystallized into what is sometimes called the "measurement problem." The basic point is to decide when we should describe the quantum dynamics with a unitary Schrödinger equation, and when we should describe it with the projection postulate of wavefunction collapse. This problem was, of course, well known to the founders of quantum mechanics and is also sometimes called the "Heisenberg cut" – where should we cut off when to describe physical systems as having classical properties versus describing them as being in a coherent quantum superposition?

This question brings us into the 1980s, where a school of thought arose that we could solve this measurement problem with the process of *decoherence*. The basic idea runs as follows: We know that in reality quantum systems are not isolated, and interact with their environment. This environment can also be described quantum mechanically. However, it is impractical – perhaps even impossible – to control every aspect of it, so it is natural to average over the dynamics of the uncontrolled and unobserved environment. Rather than use a pure quantum state, the concept of the mixed quantum state of the subsystem is then of great utility, which is a statistical mixture of pure states. Mixed states include pure quantum states, but also include classical statistical mixtures – for example, how we describe the results of a classical coin flip where the randomness is simply a reflection of our ignorance of the "microstate" of the coin. Technically mixed states are described with density matrices, which generalize the concept of state vectors. Mixed states are reviewed in Appendix B.

The idea of decoherence is that the environmental interaction degrades the quantum coherence of a pure state, converting it into more palatable classical statistical mixtures. In describing the dynamics of a system of interest with an environment, the effect of the simplest type of coupling is to add a phase to the off-diagonal density matrix elements (defined in a basis specified by the environmental coupling). This phase depends on the state of the environment, so that the dynamics of the subsystem should be averaged over the fast environmental dynamics. The result of this is that the off-diagonal density matrix elements are suppressed, resulting in a purely classical statistical mixture. The basic insight advocated by physicists such as Wojciech Zurek (1981, 1982; see Fig. 1.2) was that the environment dictates the manner of decoherence the system experiences, and quantum measurement should be seen as a kind of decoherence. At the very least, this explains why coherence is difficult to maintain in large quantum systems, or systems not isolated from their environments. This approach, fully within the standard canon of quantum mechanics, was very popular at the time.

The shortcoming of the decoherence worldview is that no measurement – in our sense of learning something about the system – ever takes place until the very end when all coherence is removed. The theory describes statistical ensembles, not measured data. The fact that decoherence theory does *not* explain individual measurement results shows it is not a fundamental theory of measurement, and

Figure 1.2 Hiking photos of Karl Kraus (left) and Wojciech Zurek (right), reclining. Reproduced with permission from wikimedia.org under CC-BY SA, and from W. Zurek.

more is required despite common impressions from the recent literature. We will see in later chapters that when the environment is monitored, a (stochastic) pure state description of the dynamics can be restored, bringing us back to a collapse process.

At the same time, Kraus (1981, 1985) and Kraus et al. (1983), continuing in the line of Sudarshan et al. (1961) and Davies (1976), were also working on the theory of open quantum systems and developed the concept of a quantum process, or quantum dynamical system. This is closely related to a dynamical description of the open quantum systems of Lindblad (1976), as well as Gorini et al. (1976). We will return to these concepts in the coming chapters. While correctly capturing the dynamics of open quantum systems, including the decoherence process, these approaches do not fully capture the informational aspects of quantum theory, which brings us into the Quantum Information age. Let us first take a tour through some of the classic experiments that helped to form our understanding of the field.

1.4 Classic Experiments: 1970–1980s

The last quarter of the twentieth century was a tremendously active period in which the seeds for another quantum revolution were sown, particularly as new experimental tools and techniques became available. These developments coincided with new theoretical ideas put forward by Bell and others, and their confluence led to experimental tests of uniquely quantum mechanical effects. While many notable experiments should be highlighted in a thorough review of quantum milestones, we will discuss only a few exemplars that have had a profound impact in the field,