A Student's Guide to the Ising Model

The Ising model provides a detailed mathematical description of ferromagnetism and is widely used in statistical physics and condensed matter physics. In this Student's Guide, the author demystifies the mathematical framework of the Ising model and provides students with a clear understanding of both its physical significance and how to apply it successfully in their calculations. Key topics related to the Ising model are covered, including exact solutions of both finite and infinite systems, series expansions about high and low temperatures, mean-field approximation methods, and renormalization-group calculations. The book also incorporates plots, figures, and tables to highlight the significance of the results. Designed as a supplementary resource for undergraduate and graduate students, each chapter includes a selection of exercises intended to reinforce and extend important concepts, and solutions are also available for all exercises.

James S. Walker received his Ph. D. in theoretical physics at the University of Washington. He is Emeritus Professor of Physics at Washington State University, and has held positions at the University of Pennsylvania and the Massachusetts Institute of Technology. He has taught the Ising model at both the undergraduate and graduate level and is the author of the highly successful undergraduate text *Physics* (Addison-Wesley) now in its fifth edition.

Other Books in the Student's Guide Series:

A Student's Guide to the Navier-Stokes Equations, Justin W. Garvin

A Student's Guide to Special Relativity, Norman Gray

A Student's Guide to Laplace Transforms, Daniel A. Fleisch

A Student's Guide to Newton's Laws of Motion, Sanjoy Mahajan

A Student's Guide to the Schrödinger Equation, Daniel A. Fleisch

A Student's Guide to General Relativity, Norman Gray

A Student's Guide to Analytical Mechanics, John L. Bohn

A Student's Guide to Atomic Physics, Mark Fox

A Student's Guide to Infinite Series and Sequences, Bernhard W. Bach Jr.

A Student's Guide to Dimensional Analysis, Don S. Lemons

A Student's Guide to Numerical Methods, Ian H. Hutchinson

A Student's Guide to Waves, Daniel A. Fleisch, Laura Kinnaman

A Student's Guide to Lagrangians and Hamiltonians, Patrick Hamill

A Student's Guide to Entropy, Don S. Lemons

A Student's Guide to the Mathematics of Astronomy, Daniel A. Fleisch, Julia Kregenow

A Student's Guide to Vectors and Tensors, Daniel A. Fleisch

A Student's Guide to Data and Error Analysis, Herman J. C. Berendsen

A Student's Guide to Fourier Transforms, J. F. James

A Student's Guide to Maxwell's Equations, Daniel A. Fleisch

A Student's Guide to the Ising Model

JAMES S. WALKER Washington State University





Shaftesbury Road, Cambridge CB2 8EA, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India

103 Penang Road, #05-06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of Cambridge University Press & Assessment, a department of the University of Cambridge.

We share the University's mission to contribute to society through the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org Information on this title: www.cambridge.org/9781009098519

DOI: 10.1017/9781009089579

© James S. Walker 2023

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press & Assessment.

First published 2023

A catalogue record for this publication is available from the British Library.

ISBN 978-1-009-09851-9 Hardback ISBN 978-1-009-09630-0 Paperback

Additional resources for this publication at www.cambridge.org/walker-sgim.

Cambridge University Press & Assessment has no responsibility for the persistence or accuracy of URLs for external or third-party internet websites referred to in this publication and does not guarantee that any content on such websites is, or will remain, accurate or appropriate.

> This book owes its existence to the Three Amigos— Betsy Walker, Janet Walker, and Jennifer Knudson. I love you!

Acknowledgements

I would like to thank the editorial and production staff at Cambridge University Press—Nicholas Gibbons, Sarah Armstrong, Elle Ferns, Jane Chan, and Reshma Xavier—for making the entire process of producing this book a most enjoyable experience. And to the students who will learn about the Ising model with this book, I hope you will come to love and appreciate it as much as I have.

About This Book

This edition of *A Student's Guide to the Ising Model* is supported by solutions to all problems and Mathematica files, available via the book's website. Please visit www.cambridge.org/walker-sgim to access this extra content.

We may update our Site from time to time and may change or remove the content at any time. We do not guarantee that our Site, or any part of it, will always be available or be uninterrupted or error free. Access to our Site is permitted on a temporary and "as is" basis. We may suspend or change all or any part of our Site without notice. We will not be liable to you if for any reason our Site or the content is unavailable at any time, or for any period.

Contents

1	The Ising Model	page 1
2	Finite Ising Systems	29
3	Partial Summations and Effective Interactions	65
4	Infinite Ising Systems in One Dimension	94
5	The Onsager Solution and Exact Series Expansions	125
6	The Mean-Field Approach	150
7	Position-Space Renormalization-Group Techniques	174
	Bibliography	211
	Index	213