



Intermediate Dynamics

This advanced undergraduate physics textbook presents an accessible treatment of classical mechanics using plain language and clear examples. While comprehensive, the book can be tailored to a one-semester course. An early introduction of the Lagrangian and Hamiltonian formalisms gives students an opportunity to utilize these important techniques in the easily visualized context of classical mechanics. The inclusion of 321 simple in-chapter exercises, 82 worked examples, 550 more challenging end-of-chapter problems, and 65 computational projects reinforce students' understanding of key physical concepts and give instructors freedom to choose from a wide variety of assessment and support materials. This new edition has been reorganized. Numerous sections were rewritten. New problems, a chapter on fluid dynamics, and brief optional studies of advanced topics such as general relativity and orbital mechanics have been incorporated. Online resources include a solutions manual for instructors, lecture slides, and a set of student-oriented video lectures.

Patrick Hamill has taught physics at San José State University for over 30 years. During that time he was honored by student organizations for teaching excellence and was named a “President’s Scholar” for his research activities in atmospheric science. He received the NASA Ames Julian Allen award for his studies of the role of polar stratospheric clouds in the formation of the ozone hole over the Antarctic. Professor Hamill has published over 100 peer-reviewed papers. He is the author of the Cambridge University Press text, *A Student’s Guide to Lagrangians and Hamiltonians*.

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Preface

Although this book begins at an introductory level, by the end of the book the student will have been exposed to all of the subject matter usually found in an intermediate mechanics course as well as a few advanced topics.

Organization

This book is divided into six parts. Part I is called “Kinematics and Dynamics.” This part (Chapters 1–4) covers kinematics in various coordinate systems, the dynamical theory of Isaac Newton, the equations of motion given by Newton’s second law, and the equations of Lagrange and Hamilton. Chapter 1 consists of a review of a few essential introductory concepts. This chapter can be skipped by well-prepared students, or assigned as reading for students who only need a quick refresher. The next chapter (Chapter 2) is called “Kinematics.” This is the traditional starting point for courses in intermediate mechanics. Here, the student is exposed to relations between acceleration, velocity, and position in Cartesian, plane polar, cylindrical, and spherical coordinates. A few simple concepts from vector analysis are introduced. A number of reasonably difficult projectile problems are included in the problems. The next chapter (Chapter 3) considers Newton’s laws. It includes a discussion on “Determining the Motion” in which the student learns techniques for integrating Newton’s second law to obtain the position as a function of time. This is done for constant forces, and for forces that are functions of time, of velocity, and of position. A short section called “Numerical Solutions” gives a flavor for the use of computational techniques in physics. The role of computers in physics is not emphasized in this course. However, I realize that many instructors want to expose their students to computational methods, so I have included a few discussions of numerical techniques. Furthermore, nearly every chapter has a number of “Computational Projects.” However, this text does not stress the role of computers in physics because I find that teaching the traditional material of intermediate mechanics takes most of two semesters and does not give enough time to delve into computational physics. Furthermore, many universities have included a computational physics course in the undergraduate physics curriculum. The next chapter (Chapter 4) is called “Lagrangians and Hamiltonians.” I think it is important for physics students to be exposed to these concepts early on in their study of mechanics. The Lagrangian is presented, at first, as a simple technique for generating the equation of motion. Later in the chapter, I go through a derivation of the Lagrange equations using the calculus of variations. This section need not be covered if the instructor feels it is too advanced. The chapter ends with a discussion of the Hamiltonian and Hamilton’s equations. There are several reasons why I chose to present the Lagrangian early in the course, perhaps the most important being that it gives the student a simple (almost “cookbook”) technique for obtaining the equations of motion for a complicated dynamical system. For example, the Lagrangian technique allows one to determine the equations of motion for a double pendulum, for a spherical pendulum, or for coupled oscillators. More importantly, it allows one to introduce the concepts of generalized momentum and ignorable

coordinates and leads to the relation between conservation laws and symmetries. Furthermore, it lets the student know that this course is not simply a rehash of concepts learned in introductory physics.

Part II (Chapters 5–8) is denoted “Conservation Laws” in which conservation principles are treated in depth. These chapters cover the conservations of energy, linear momentum, and angular momentum (Chapters 5, 6, and 7), followed by a short chapter (Chapter 8) on the relation between symmetry and conservation laws. Chapter 5, on the conservation of energy, discusses potential energy and the use of energy diagrams. Potential energy naturally leads to a discussion of the gradient of a scalar field. There is a section on the way the “Del” operator can be expressed in cylindrical and spherical coordinates; this allows one to discuss coordinate transformations in general. (I believe that introducing concepts from vector calculus as required by the physics is more effective than stuffing all of the vector concepts into a single introductory chapter.) Chapters 6 and 7, on the conservation of linear and angular momentum, cover the usual topics (rockets, collisions, etc.) as well as some less usual topics such as the fact that angular momentum is an axial vector.

Part III is called “Gravity” and consists of two chapters. The first one (Chapter 9) deals with Newtonian gravity and is an introduction to field theory. The study is limited to considerations of the gravitational field, because field theory is treated exhaustively in courses on electromagnetism. Additional vector concepts are introduced here and the student is exposed to Gauss’s law and the equations of Poisson and Laplace. I felt that a chapter on gravity would be incomplete if Einstein’s contributions were ignored. The topic is rather forbidding from a mathematical point of view, but I attempted to present it in a way that would make sense to students at this stage of their development. Nevertheless, I expect that many instructors will prefer not to discuss this admittedly superficial analysis, so I have labeled the section “Optional.” The next chapter (Chapter 10) deals with central force motion in a gravitational field, as illustrated by the Kepler problem. Also considered is the stability of circular orbits, showing the student how to deal with small perturbations. Specifically, we imagine a comet striking a planet in a previously perfectly circular orbit and analyze the planet’s subsequent motion to determine stability conditions and the frequency of radial oscillations.

Part IV (Chapters 11–14) is called “Oscillations and Waves.” In Chapter 11 damped and driven harmonic oscillators are treated in depth. A rather thorough discussion on how to solve second order differential equations is included here. Coupled oscillators and normal modes are considered. Chapter 12 is on the motion of a pendulum. We begin with the motion of a simple pendulum of arbitrary amplitude and introduce elliptic integrals. Next we consider the physical pendulum, centers of oscillation and percussion, the spherical pendulum, and the conical pendulum. To spend a whole chapter on the pendulum may seem excessive, but it is a simple, easily visualized physical system that allows one to introduce many useful mathematical techniques without having to spend time explaining the motion. The next chapter in this part (Chapter 13) is an introduction to wave motion. This topic is not considered in great detail because it is treated extensively in the undergraduate electromagnetic theory class. Nevertheless, the student will receive a reasonably complete overview of mechanical waves. Sound waves require knowledge of fluid dynamics and are left to Chapter 19. The last chapter in Part IV is an analysis of small oscillations. It is rather advanced and the chapter is denoted as optional.

Part V (Chapters 15–17) is called “Rotation.” Portions of the material in this part may be too advanced for some classes, but the instructor will probably want to cover Chapter 15 and some topics in rotational kinematics and dynamics. The first chapter in this part (Chapters 15) is called “Accelerated Reference Frames” in which we (mainly) consider motion on the surface of the rotating Earth. Coriolis forces and the Foucault pendulum are treated. Perturbation theory is used to solve these problems. Chapter 16 (on rotational kinematics) introduces orthogonal transformations. We obtain the Euler angles and consider Euler’s theorem. The following chapter (Chapter 17) on rotational dynamics introduces the inertia tensor and some simple methods from tensor analysis.

The last part of the book is called “Special Topics” and consists of four chapters (18–21). The first chapter in this part (Chapter 18) is a fairly advanced study of statics, including a discussion of d’Alembert’s principle and the concept of virtual work. The next chapter (Chapter 19) is called “Fluid Dynamics and Sound Waves.” Logically, this material could be treated immediately after Chapter 13, but the mathematics gets fairly complicated so I placed it near the end of the book and marked it as “optional.” The instructor may wish to cover the section on hydrostatics and skip the rest of the chapter. The next chapter (Chapter 20) “The Special Theory of Relativity” is an introduction to special relativity, and the final chapter (Chapter 21) “Classical Chaos” is a brief introduction to chaos. These two chapters are simply intended to give the student a flavor of these interesting subjects.

A One-Semester Course

Many instructors will find that the intermediate mechanics course in their department has been reduced to one semester. In such a situation it is impossible to cover all of the material in this book. From a personal perspective I feel that the essential material is covered in Parts I and II and Chapters 11, 15, and 16.

Exercises and Problems

Learning physics requires doing physics, so I have included a large number of “exercises.” These are found at the end of nearly every section. Most of them are fairly easy. Some are merely “plug-ins” to get the student to look at a formula and (hopefully) to think about it. Others ask the student to fill in the missing steps in a derivation. A few require a bit of clever thinking. Nearly all have answers given. I hope that students studying this book will solve every one of these exercises. At the end of each chapter is a collection of problems that are of the degree of difficulty to be expected from a course at this level. Many of these will require significant effort on the part of the student. However, I believe that a student who has read the chapter and worked the exercises will be prepared to attack the problems.

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