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Excerpt
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Part I

Kinematics and Dynamics

1 A Brief Review of Introductory Concepts

You already know a lot of physics and quite a bit of mathematics. You have been exposed to introductory courses in Mechanics, Electromagnetism, Thermodynamics, and Optics. You are now beginning your studies of these topics in a much more profound and rigorous manner. I hate to tell you this, but you are expected to know those concepts from the introductory courses! I know how easy it is to forget definitions and mathematical relationships if you are not using them all the time. So this chapter is a brief review of some of the concepts from your introductory mechanics course that you will be using in this intermediate level course. (I have included only those concepts that are absolutely necessary.) If the brief explanations in this chapter are not sufficient, please go back to your introductory physics textbook and review the material there. The standard introductory physics texts are well written and contain many instructive figures and diagrams. It is a good idea to refer to that text whenever you are exposed to the same material on a more advanced level.[1]

Just as in introductory physics, you will begin the upper-division physics sequence with this course in classical mechanics. This is the most beautiful of all physics courses (at least in my opinion!). It is also probably the most useful of all physics courses as it is the basis of essentially all advanced physics. The brilliant physicists who developed Quantum Mechanics, Relativity, Statistical Mechanics, etc., were experts in Classical Mechanics. The Lagrangian (which you will study in Chapter 4) is the basis of Elementary Particle Physics and the Hamiltonian (also in Chapter 4) is fundamental in Quantum Mechanics.

As you may recall, the mechanics section of your introductory physics book covered the following topics:

- Kinematics
- Newton's second law
- Work and energy
- Momentum
- Rotational motion.

We now very briefly review some concepts from each of these items.

1.1 Kinematics

Kinematics is the study of motion. Essentially, kinematics involves determining the relationships between position, velocity, acceleration, and time.

[1] If you are a particularly well-prepared student and feel that you know the material in this chapter, I suggest that you go to the end of each section where you will find a few exercises. If you can solve them, then skip to the next section, but if you feel uncertain or even somewhat confused, read the section. You might also try solving a few of the problems at the end of the chapter.

Position is denoted by the vector[2] $\mathbf{r}$ and the change in position (or displacement) can be written as $\Delta\mathbf{r}$.

Velocity is defined as the displacement with respect to time, so the average velocity is given by

$$\langle \mathbf{v} \rangle = \frac{\Delta\mathbf{r}}{\Delta t},$$

where Δt is the time during which the object had a displacement $\Delta\mathbf{r}$.

As the time interval becomes very small, we replace the *difference* (represented by Δ) with the *derivative* and write

$$\mathbf{v} = \frac{d\mathbf{r}}{dt}. \tag{1.1}$$

The change in velocity with respect to time is called the acceleration and is given by

$$\mathbf{a} = \frac{d\mathbf{v}}{dt}. \tag{1.2}$$

We can use the definitions of acceleration and velocity to write the inverse relation:

$$\int d\mathbf{v} = \int \mathbf{a}dt.$$

Integrating we get

$$\mathbf{v} = \int \mathbf{a}dt + \text{constant}.$$

Integrating again,

$$\mathbf{r} = \int \mathbf{v}dt + \text{constant}.$$

These are vector relationships and are valid in any coordinate system. In Chapter 2 you will find the relations between acceleration, velocity, and position in various coordinate systems.

1.1.1 Motion in a Straight Line at Constant Acceleration

For motion in a straight line,[3] we do not need to use vector notation. Let the position to be represented by x. If the acceleration is constant the integrals above lead to the familiar relations

$$v(t) = at + v_0, \tag{1.3}$$

and

$$x(t) = \frac{1}{2}at^2 + v_0 t + x_0, \tag{1.4}$$

where v_0 and x_0 are the initial velocity and the initial position.

You probably memorized Equations (1.3) and (1.4) in your introductory physics course. But don't forget that they are only valid if the acceleration is constant. In this course, we shall frequently be concerned with nonconstant accelerations and these relations cannot be used. The appropriate relations are derived in Chapter 3.

Two other useful relations can be obtained from Equations (1.3) and (1.4). Solving one equation for t and substituting it in the other yields

$$2a(x - x_0) = v^2 - v_0^2.$$

[2] In this book vectors are represented in *bold face* ($\mathbf{r}$), whereas scalars are represented by *italics* (r).
[3] Motion in a straight line is an example of one-dimensional motion.

Similarly, if we solve one equation for a and substitute it in the other equation we obtain

$$x - x_0 = \frac{1}{2}(v + v_0)t.$$

Again I emphasize that these relations are valid only if the acceleration is constant.

Comment In this book you will find a large number of "exercises." They are not difficult. They are intended to give you a chance to review and understand the concepts in the preceding section. It would be a good idea to work out the solution for each exercise as you read through the text. At the end of each chapter you will find a set of problems that are significantly more difficult than the exercises. However, if you solve the exercises, you will be well prepared for solving the problems. Many students find working the exercises to be a good preparation for the examinations.

Exercise 1.1

You were driving your new Ferrari at 62 mph (= 100 km/h) when you spotted a police car. Naturally, you hit the brakes. You slowed to 31 mph, covering a distance of 50 m. (a) What is your constant acceleration? (b) How much time did it take to slow to 31 mph? Answers: (a) -5.79 m/s^2, (b) 2.4 s. ■

1.2 Newton's Second Law

The study of the relation between the forces acting on a body and the motion of the body is called dynamics. In your introductory course you were exposed to dynamics in the form of Newton's second law. That law states that a body of mass m acted upon by a force $\mathbf{F}$ will accelerate at

$$\mathbf{a} = \mathbf{F}/m.$$

This relationship is usually remembered as

$$\mathbf{F} = m\mathbf{a}. \tag{1.5}$$

As we will discuss in Chapter 3, this form of Newton's second law is only valid if the mass is constant. The relation $\mathbf{F} = m\mathbf{a}$ is usually applied to a force that is acting on a *point mass* (often referred to as a "particle"). However, it can also be applied to an extended rigid body. Then $\mathbf{a}$ is defined as the acceleration of the center of mass.

Worked Example 1.1 Determine the acceleration of a block of mass m sliding down an inclined plane of angle θ. Assume the coefficient of sliding friction is μ. See Figure 1.1.

Solution The forces acting on the block are gravity mg (downwards), the normal force N (perpendicular to the plane), and the frictional force μN (parallel to the plane). These forces are illustrated in Figure 1.1(a). The *free-body diagram* with all of the forces acting at the center of mass of the block is shown in Figure 1.1(b). In a problem such as this one, it is convenient to assume the axes are parallel to and perpendicular to the surface of the plane as indicated in Figure 1.1(c).

There is no acceleration perpendicular to the plane so the net force in that direction must be zero. Consequently, $N = mg\cos\theta$. The net force down the plane is $F_d = mg\sin\theta - \mu N$. The acceleration of the block is

$$a = \frac{F_d}{m} = g\sin\theta - \mu g\cos\theta.$$

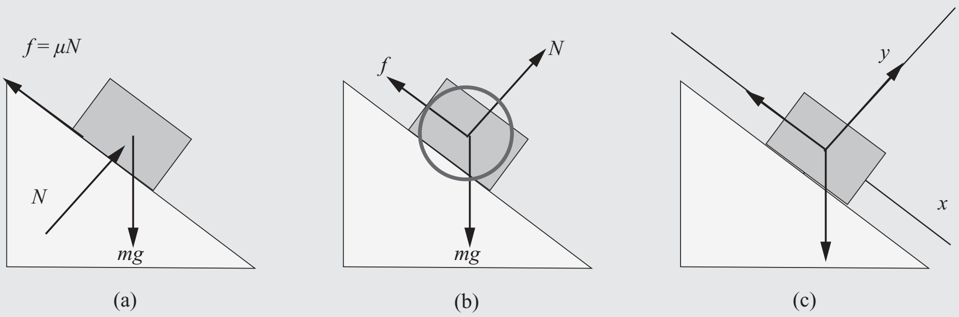

Figure 1.1 A block sliding down an inclined plane with friction. Sketch (a) shows the forces acting *on* the block. Sketch (b) shows the free-body diagram. Sketch (c) shows the x- and y-axes inclined so that they are parallel and perpendicular to the plane.

Exercise 1.2

Two blocks of mass M_1 and M_2 are tied together. They are sitting on a smooth frictionless surface as shown in Figure 1.2. A force F is applied to the free string attached to M_1. What is the tension in the string between the two blocks? Answer: $T = M_2F/(M_1 + M_2)$. ■

Exercise 1.3

A block of mass 25 kg is held in place on an inclined plane of angle 30° as shown in Figure 1.3. The coefficient of static friction is 0.4. (a) Draw the free-body diagram. What forces act on the block? (b) What is the tension in the string? (c) If the string is cut, what is the acceleration of the block? Answers: (b) $T = 122.5$ N, (c) $a = 1.51$ m/s^2. ■

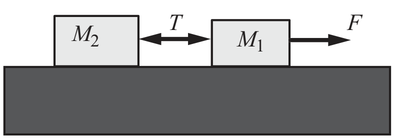

Figure 1.2 Two blocks on a smooth frictionless surface connected by a massless string.

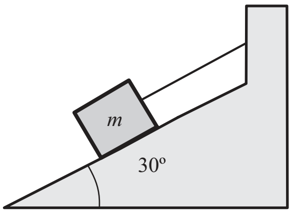

Figure 1.3 A block on an inclined plane.

1.3 Work and Energy

This section outlines some aspects of work and energy. An in-depth study is presented in Chapter 5.

Let us begin with the concept of work. Imagine pushing a box along a horizontal surface, such as a tabletop. The force you are applying can be denoted by **F**. As you might expect, there will be opposing forces such as friction, air resistance, etc., but for now we are interested only in the force *you* are exerting. If you push the box from $\mathbf{r}_1$ to $\mathbf{r}_2$, the *work* you do on it is defined to be

$$W = \int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{F} \cdot d\mathbf{r}. \tag{1.6}$$

Power is defined as the work done per unit time. Suppose a force **F** acts on a particle for an infinitesimal time interval, from t to $t + dt$. The work done during this interval is $dW = \mathbf{F} \cdot d\mathbf{r}$. Therefore, the power is

$$P = \frac{dW}{dt} = \frac{\mathbf{F} \cdot d\mathbf{r}}{dt}.$$

Since $d\mathbf{r}/dt \equiv \mathbf{v}$, another expression for power is $P = \mathbf{F} \cdot \mathbf{v}$.

When we do work on an object, we usually change its *energy*. The mechanical energy of an object is either kinetic energy (energy of motion) or potential energy (energy of position). For a particle of mass m moving with speed v the kinetic energy (denoted by T) is given by

$$T = \frac{1}{2}mv^2. \tag{1.7}$$

The potential energy of an object of mass m raised a height h above the surface of the Earth is given by

$$V = mgh. \tag{1.8}$$

We shall often be interested in the potential energy of a stretched or compressed spring. The potential energy of such a system is given by

$$V = \frac{1}{2}kx^2, \tag{1.9}$$

where k is the spring constant and x represents the amount the spring is stretched or compressed.

There is a very important relationship between potential energy and force. Consider Equation (1.8), but express it as $V = mgz$, where z is the height above some reference surface and V is the potential energy of an object raised to height z in the gravitational field near the surface of the Earth. The force of gravity on the object is mg. Note that

$$F = \frac{dV}{dz} = \frac{d}{dz}(mgz) = mg.$$

Similarly, the force exerted by a spring is $F = kx$ and the potential energy stored in a spring is $V = \frac{1}{2}kx^2$. Consequently,

$$F = \frac{dV}{dx} = \frac{d}{dx}\frac{1}{2}kx^2 = kx.$$

Thus it would seem that force is the derivative of potential energy with respect to position. That is not quite correct. For one thing, the sign on F is wrong and for another thing, we obtained a scalar. That is, in our examples we just evaluated the magnitude of the force, but we know that force is a vector. The correct relationship between force and potential energy is

$$\mathbf{F} = -\nabla V. \tag{1.10}$$

This is discussed in detail in Chapter 5, but it is useful at this stage to remember that force and potential energy are related to one another. Forces that can be derived from a potential energy as given by Equation (1.10) are called conservative forces.

If the only work done on a system is due to conservative forces the sum of kinetic energy and potential energy is constant ($T + V =$ constant). This is called the law of conservation of energy.

Exercise 1.4

A rock is thrown upward from the top of a 30 m building with a velocity of 5 m/s. Determine its velocity (a) when it falls back past its original point, (b) when it is 15 m above the street, and (c) just before it hits the street. Answer: (a) −5 m/s, (c) 24.76 m/s. ■

Exercise 1.5

A horse drags a 100 kg sled a distance of 4 km in 20 min. The horse exerts one horsepower, of course. What is the coefficient of sliding friction between the sled and the ground? Answer: $\mu_k = 0.23$. ■

1.4 Momentum

In Chapter 6 you will encounter a detailed study of linear momentum. Here I just want to remind you of a few facts from introductory mechanics.

A moving particle is characterized by having a particular *momentum*. When we use the term "momentum" we usually are referring to the linear momentum, not to be confused with the angular momentum, which we will define in a little while.

Momentum is a vector defined as mass times velocity and is denoted by the letter **p**. Thus,

$$\mathbf{p} = m\mathbf{v}. \tag{1.11}$$

If the mass of a body is constant, the time derivative of the momentum is

$$\frac{d\mathbf{p}}{dt} = \frac{d(m\mathbf{v})}{dt} = m\frac{d\mathbf{v}}{dt} = m\mathbf{a} = \mathbf{F}.$$

In Equation (1.5) we wrote $\mathbf{F} = m\mathbf{a}$ and called it Newton's second law. But that is only valid if the mass is constant. We now appreciate that a more general expression of Newton's second law is

$$\mathbf{F} = \frac{d\mathbf{p}}{dt}. \tag{1.12}$$

This relationship is valid even if the mass is not constant. In fact, it is the most general statement of Newton's second law.

The force **F** appearing in Newton's second law is the net or total *vector sum* of all forces acting on the body. Consequently, we appreciate that if the net force is zero, the time derivative of the momentum is zero. That is, the momentum of the body is constant. This is called the law of conservation of momentum.

Exercise 1.6

A 1500 kg car traveling East at 40 km/h turns a corner and speeds up to a velocity of 50 km/h due North. What is the change in the car's momentum? Answer: 26 700 kg m/s at 38.7° West of North. ■

1.5 Rotational Motion

1.5.1 Rotational Kinematics

The motion of a rigid body rotating about a *fixed axis* is mathematically identical to one-dimensional linear motion. Recall that kinematics is a study of the relationship between position, velocity, acceleration, and time. Rotational kinematics deals with angular position (θ), angular velocity (ω), and angular acceleration (α), where angular velocity is defined by

$$\omega \equiv \frac{d\theta}{dt},$$

and angular acceleration is

$$\alpha \equiv \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}.$$

You will discover in Chapters 7, 16, and 17 that rotational motion can be very complicated. To keep things simple for the moment, consider the special case of a *symmetrical* body rotating about a *fixed* axis, such as the wheel illustrated on the left side of Figure 1.4.

For a fixed, stationary axis, the center of the wheel is at rest. All other points are moving in circles around it. If you looked straight down the axis you would see the circle shown on the right side of Figure 1.4. Point P is on the rim of the wheel. The angular position of P is given by the angle between some fixed line and the radius vector to P. If the wheel is turning, after a time dt point P will have moved a distance ds to P'. Recall from geometry that $ds = r d\theta$, where $d\theta$ (in radians of course!) is the angle subtended by the arc $PP' = ds$. Point P moves with speed

$$v = \frac{ds}{dt} = \frac{r d\theta}{dt} = r\omega.$$

The speed of point P is called the "tangential speed" because instantaneously P is moving tangent to the rim of the wheel. It is sometimes convenient to write the tangential speed as v_T. Then

$$v_T = r\omega. \tag{1.13}$$

Taking the time derivative of v_T yields the tangential acceleration a_T,

$$a_T = \frac{dv}{dt} = \frac{d}{dt}(r\omega) = r\frac{d\omega}{dt},$$

where we used the fact that r is constant. But $\frac{d\omega}{dt} = \alpha$, so

$$a_T = r\alpha. \tag{1.14}$$

This is the relationship between the tangential acceleration a_T and the angular acceleration α.

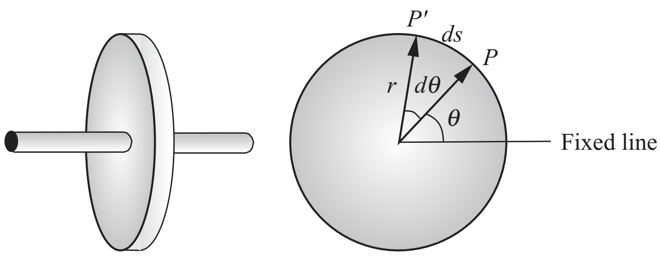

Figure 1.4 A wheel mounted on a fixed axis. The wheel is allowed to rotate but not to translate.

It is often convenient to express angular velocity and angular acceleration as vectors ($\boldsymbol{\omega}$) and ($\boldsymbol{\alpha}$) directed along the axis of rotation.

Exercise 1.7

A wheel initially spinning at $\omega_0 = 50.0$ rad/s comes to a halt in 20.0 seconds. Determine the constant angular acceleration and the number of revolutions it makes before stopping. Answers: -2.5 rad/s^2, 79.6 rev. ■

1.5.2 Rotational Dynamics

Rotational dynamics is the analysis of the motion of a body subjected to external *torques.*

Consider a body constrained to rotate about a fixed axis, as shown in Figure 1.5. I drew the body in the shape of a plane lamina for simplicity. Let a force **F** act on the body at a point on its rim. Let us assume (again for simplicity) that **F** is perpendicular to the axis of rotation. The point of application of **F** is specified by the vector **r** whose origin is at the axis of rotation.

The body cannot accelerate linearly because the axis is fixed. The applied force causes the body to rotate about the axis. The tendency of a force to cause a rotation is called the *moment* of the force, or more commonly, the *torque.* Just as you can think of a force as a pull, you can think of a torque as a twist.

The ability of a force to produce a rotation depends not only on the *magnitude* of the force, but also on its *direction* and on the *location* of the point where the force is applied to the body.

To define torque draw a line having the direction of the force and passing through the point where the force is applied. This is called the "line of action of the force." (See Figure 1.6.) Next draw a line that starts at the axis of rotation and intersects the line of action at a 90° angle. This

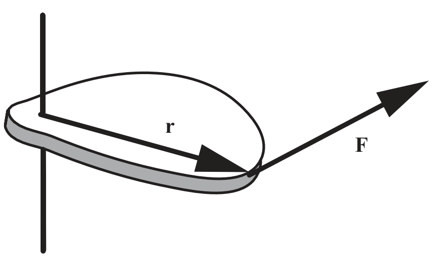

Figure 1.5 Illustration of torque. The laminar body is free to rotate around the fixed axis of rotation. The vector **r**, with origin at the axis, specifies the point of application of the force.

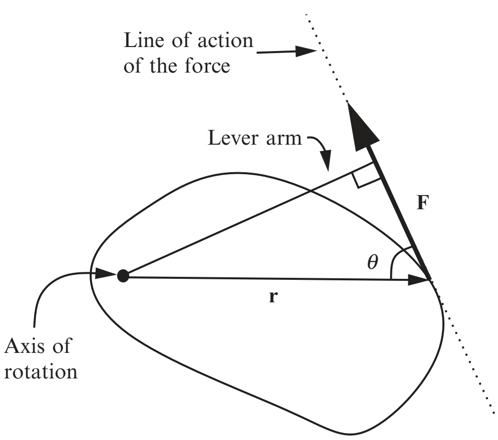

Figure 1.6 Definition of lever arm. The axis of rotation is perpendicular to the plane of the figure.