Quantitative Enterprise Risk Management

This well-balanced introduction to enterprise risk management (ERM) integrates quantitative and qualitative approaches and motivates key mathematical and statistical methods with abundant real-world cases – both successes and failures. Worked examples and end-of-chapter exercises support readers in consolidating what they learn. The mathematical level, which is suitable for graduate and senior undergraduate students in quantitative programs, is pitched to give readers a solid understanding of the concepts and principles involved without diving too deeply into more complex theory. To reveal the connections between different topics, and their relevance to the real world, the presentation has a coherent narrative flow, from risk governance, through risk identification, risk modelling, and risk mitigation, capped off with holistic topics – regulation, behavioural biases, and crisis management – that influence the whole structure of ERM. The result is a text and reference that is ideal for graduate and senior undergraduate students, risk managers in industry, and anyone preparing for ERM actuarial exams.

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QUANTITATIVE ENTERPRISE RISK MANAGEMENT

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To Phelim and Ruth

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Preface

Enterprise risk management (ERM) is an increasingly popular university course within actuarial science, finance, risk management, and business programmes. The evolution of ERM over the past couple of decades has proceeded along two lines: the qualitative approach, which covers, for example, governance frameworks, risk identification procedures, and ERM processes; and the quantitative approach, which takes a more technical, mathematical perspective on risk modelling and risk measurement.

The aim of this book is to bring the qualitative and quantitative approaches together in a coherent and integrated text, at a level suitable for graduate and upper-level undergraduate students in mathematical and engineering sciences. It is also designed to be a useful self-study aid for students working towards professional actuarial exams, such as those offered by the Society of Actuaries towards the Chartered Enterprise Risk Analyst (CERA) certification.

This book has been a work in progress for many years. The rapidly changing ERM environment led to significant updates before publication to deal with changes in regulations and in risk management practice. As we write this preface, the global COVID-19 pandemic is ongoing, and it is difficult to predict how this will change risk management, even in the short term. The regulation of financial institutions is constantly evolving, and while we have strived to keep our presentation of the material current, one is always best advised to consult the regulatory documents themselves for the most detailed and up-to-date requirements.

There are several decisions that we made in writing the text, which may not meet with universal agreement. There are no exact start and end dates for the crisis that shook the global financial system in the first decade of this century. We have referred to it as the global financial crisis of 2007–8 throughout, in full recognition that this period omits the failures of many subprime lenders in 2006 and concerns about the potential for sovereign defaults that extended into

2009 and beyond. We have generally used LIBOR (London Interbank Offered Rate) as the underlying floating rate in our examples of interest rate derivative securities, acknowledging its impending obsolescence.

There are also things that we would have done differently if we had the time to start over. Re-reading the final draft, we felt that more attention should have been given to climate change, which presents not only a serious risk to all businesses but an existential threat to humanity.

This book contains over 175 exercises. The exercises are of several different types, including qualitative discussions, mathematical calculations and derivations, and computational exercises on real data sets. We encourage readers to attempt as many of these exercises as possible, as this is the only path to true mastery of the material. We are particularly grateful to the Society of Actuaries for allowing us to reproduce some problems from their past professional examinations. Whenever we have done so, it is indicated explicitly in the text. We have, on occasion, made minor modifications in the wording of these problems so that they fit more closely with the presentation in the text.

The material in this book is suitable for a sequence of two one-semester courses. Chapters 1–6 provide a solid foundation in qualitative and quantitative ERM; Chapters 7–10 could round off the first course, adding context with a slight focus on market risk and banking. In the second course, Chapters 11–20 provide a more in-depth look at some specific risks, and in the final chapters, the material returns to the top-down perspective, considering institutional issues that can promote or undermine the organization's risk management.

Chapter Summary

We begin, in Chapter 1, with an introduction to ERM. We present a historical view of the evolution of the subject and show how modern ERM has its origins in the individual areas of risk management that preceded it. We discuss ERM as an ongoing process and an integral part of the firm. We introduce the stages of the ERM cycle, including risk identification and analysis, risk evaluation, and risk treatment, each of which is to receive more detailed treatment later on in the book.

Chapter 2 presents a classification and description of the major types of risk that may be faced by an organization. We cover both external risks (those that arise from outside the organization) and internal risks (risks that are generated from within the organization). External risks are divided into economic, political, and environmental risks, and internal risks into operational and strategic risks. We also look at reputational risk, which may be either

internally or externally generated. We consider several examples of how risks have arisen in high profile cases, illustrating as well how different types of risks can be driven by the same event.

Chapter 3 presents an introduction to risk measures, with a focus on Value at Risk (VaR), and Expected Shortfall, the two most commonly employed measures in quantitative risk management in practice. We study several important issues, including key properties that risk measures may possess (or fail to possess), that is, the axioms for a coherent risk measure, and computational issues, including estimating risk measures and how to calculate standard errors for risk measures estimated using Monte Carlo simulation.

Chapter 4 discusses frequency-severity analysis, a key workhorse in many real-world applications of quantitative risk management. In these models, the losses incurred are modelled as a compound random variable: the sum of a random number of independently and identically distributed losses. We discuss several common distributions used for both the frequency variable (the number of losses) and the severity distribution (the amount of each loss). Basic analytical results that can be used to calculate moments for frequency and severity models are presented, as are numerical methods needed to compute more complicated functions of loss distributions, such as risk measures.

Chapter 5 presents the basics of the application of extreme value theory to risk management. Extreme value theory provides us with a collection of tools that can be used to supplement traditional statistical analysis with asymptotic and statistical methods that focus on the extreme tails of loss distributions. We study both the block maxima and points-over-thresholds approaches, including the fundamental limiting results and corresponding statistical techniques. Approximation formulas for VaR and Expected Shortfall derived from the asymptotic limit theorems are presented.

Chapter 6 introduces copulas, the fundamental tool for modelling the dependence between loss random variables. We discuss copula basics, including Sklar's theorem, bounds for dependent risks, and basic implicit and explicit copulas. We also discuss measures of dependence and, in particular, how rank dependence can provide more information than the ubiquitous Pearson correlation coefficient. In keeping with our interest in the tails of loss distributions, we present measures of tail dependence. The chapter concludes with a discussion of the construction and estimation of copulas.

Chapter 7 discusses stress testing and scenario analysis, in which the behaviour of a portfolio under a collection of possible future scenarios is analysed. We consider important properties that stress scenarios should have, and the use and limitations of stress tests in practice. Different approaches for generating stress scenarios, including top-down and bottom-up approaches,

and reverse stress testing are discussed. Finally, we discuss regulatory stress tests and consider some examples of past failures and successes of real-world stress tests.

Chapter 8 considers mathematical models commonly employed in the analysis of market risk. We begin by presenting an analysis of the behaviour of some series of equity returns, as an example of how to identify important empirical properties that should be reproduced by the mathematical models. We then describe three models that are commonly used to model asset returns in practice: the independent lognormal model, the GARCH (generalized autoregressive conditionally heteroscedastic) model, and the regime-switching lognormal model. We discuss the properties of the different models and the extent to which they reproduce our empirical observations about stock returns. We also study methods for estimating the model parameters and how to choose between the different models in practice.

Chapter 9 looks at techniques employed in the calculation of short-term portfolio risk. We review analytical approximations such as the delta-normal and delta-gamma-normal approaches, and then we discuss historical simulation. We conclude with a discussion of backtesting risk measure estimates.

Economic scenario generators (ESGs) for the modelling of risk over long time horizons are the subject of Chapter 10. We look both at models with a cascade structure, such as the Wilkie model, and vector autoregressive models. Important properties of ESGs are discussed, and we consider some of the challenges of model and parameter selection for long-term forecasting, such as structural breaks in the historical data. The application of ESGs to problems in risk management is illustrated through an example of an employer-sponsored pension plan.

Chapter 11 presents an overview of interest rate risk and the techniques for its management. We begin with a brief review of the term structure of interest rates and the mathematics of interest. We then proceed to a discussion of interest rate derivative securities. This is followed by a study of measures of interest rate sensitivity, including duration, convexity, and key rate duration, and their application to the mitigation of interest rate risk. The chapter concludes with an empirical discussion of the dynamics of the interest rate curve using principal component analysis.

Chapter 12 studies mathematical models for credit risk. We begin with an overview of credit sensitive instruments and the types of credit risk faced by financial market participants. We then discuss the three key components of mathematical credit risk models: probability of default, loss given default, and exposure at default. Models for the default of individual borrowers, including structural and reduced form models, are then presented and the role of credit ratings is also discussed. Finally, we discuss portfolio credit risk models, which

dramatically illustrate the importance of the dependence modelling techniques discussed in Chapter 6.

Chapter 13 looks at liquidity risk. Access to highly liquid assets is a critically important consideration for firms that need to meet their short-term cash needs, either in normal operations or in crises. We distinguish between funding and market liquidity, as well as between systemic and idiosyncratic liquidity risk, and illustrate how cash flow scenario tests can be used to identify and mitigate liquidity risks. Liquidity adjusted risk measures and their application in banking are discussed. The chapter concludes with a discussion of emergency plans for managing extreme and unexpected liquidity shocks.

Chapter 14 presents a detailed discussion of model risk and governance. We discuss different sources of model risk, including defective models, inappropriate applications, and inadequate or inappropriate interpretation of the results. We review the model life cycle and quantitative approaches to measuring model and parameter uncertainty. Finally, we discuss model governance and methods for mitigating model risk.

Chapter 15 covers the use of options and other derivative securities for risk mitigation. We consider several types of risk including equity prices, interest rates, credit, exchange rates, and commodity prices and discuss the use of derivatives for managing the risk of each.

Chapter 16 studies risk transfer. We first consider general principles of risk transfer and then present several relatively recent innovations in the transfer of insurance risk. In particular, we study the use of captive insurance companies and the securitization of insurance risk, including Catastrophe (Cat) Bonds, pandemic bonds, and longevity derivatives.

Chapter 17 discusses regulation of financial institutions and, in particular, the Basel Accords in the banking industry and the Solvency II regime for regulating insurers in the European Union (EU). We do not attempt a comprehensive summary of all the different capital charges in each regime, but summarize the main aspects of each regime and focus on the underlying principles for the regulation of financial institutions.

Chapter 18 discusses capital allocation and performance measurement. Once the risk of a portfolio has been calculated, an important next step is to understand how that risk comes about; in particular, is it possible to assign to different portfolio constituents a quantitative measure of how much they contribute to overall portfolio risk? This has several applications in ERM, including assigning capital to different business units, hedging, and performance measurement. We present many different methods that have been proposed in the literature for the allocation of capital based on a quantitative risk measure. We also discuss characteristics that a capital

allocation methodology should have and several real-world considerations in the application of capital allocation.

Over the past few decades, behavioural finance has grown in importance, highlighting several of the shortcomings of neoclassical economic and financial theory and studying how humans make financial decisions in the real world. In Chapter 19, we discuss the implications of behavioural finance for ERM. In particular, we study behavioural biases and how they can lead to risk management failures. We also present an introduction to Cumulative Prospect Theory, which provides a quantitative model of decision-making that reflects some universal cognitive biases.

Chapter 20 presents a discussion of crisis management, focussing on how to prepare for, and respond to, crises. We present several examples of good and bad responses to crises and discuss the impact of corporate structure and ethics on crisis response.

The appendix collects some material on probability and statistics that should be familiar to readers of the book. It is there for convenience and should not be mistaken for a comprehensive introduction to the subject matter.

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Drafts of chapters of this book have been used in courses on risk management that have been taught at the University of Waterloo over the past few years, and we have received many helpful comments and suggestions from students and colleagues. Particular thanks go to Carole Bernard, Phelim Boyle, Jessica Dang, Ben Feng, Charlie Ford, Marius Hofert, Joseph Hyun Tae Kim, Johnny Siu-Hang Li, Ken Seng Tan, Ruodu Wang, Saisai Zhang, and Xiaobai Zhu. John Hardy, David Hardy, and Peter Hardy all provided insight into real-world project management. Special thanks go to Felice Liang, whose research assistance was invaluable during the final months of writing. All remaining errors in the book are, of course, our responsibility.

The Department of Statistics and Actuarial Science at the University of Waterloo has provided us with a wonderful environment in which to teach, collaborate, and study. One of the most painful aspects of the recent pandemic has been our inability to interact in person with our colleagues and students.

As always, our most profound debt of gratitude goes to our families. Completing a project of this size can be an imposition on life at home in the best of times, and is especially challenging during a pandemic. Mary thanks Phelim for his unstinting support, generosity, and forbearance. David thanks Ruth for her love and support, and Andrew, Sean, and Sofia for bringing joy and excitement to every day.