



INDEX

- absolute value, 14
 - basic properties, 14
- analysis, 234
- arcsine, 133
- arithmetic mean, 9
- Arithmetic-Geometric Mean Inequality, 9, 10
 - for four numbers, 22
 - for three numbers, 22
 - general, 99, 126
- arrangement, 207, 209
- associativity, 44
- average, 9
- axiom, 43

- Basic Facts (for inequalities), 7
- Bernoulli's Inequality, 102
- Bézout's Identity, 173
- bijection, 128
- binary representation, 118
- binomial coefficients, 216
 - symmetry, 216
- Binomial Theorem, 215
- bounded (function), *see* function

- Cantor's Theorem, 149
- cardinality, 128, 139, 140, 151
- chairperson identity, 216
- closure, 44
- codomain, 36
- combination, 211
- combinatorics, 207
- commutativity, 44
- completeness axiom, 49

- complex exponential function, 293
- complex number, 266–268
 - absolute value, 273
 - argument, 285
 - conjugate, 273
 - imaginary part, 268
 - modulus, 273
 - polar representation, 285
 - real part, 268
 - roots, 288, 289
 - square roots, 280, 281
- complex plane, 276
- complex polynomial equations, 292
- composite number, 18
- composition, *see* function
- congruence modulo n , 196
- conjunction, 69, 71
- connective, 67, 68
- continuity, 254, 255
 - on intervals, 256
- contradiction, *see* proof
- contrapositive, 81
- convergence, *see* limit
- coprime, 168
- counting numbers, 16
- cubic equation, 266

- De Moivre's Theorem, 288
- De Morgan's Laws, 34
- derangement, 229
- derivative, 258
- diagram
 - arrow, 37
 - bubble, 37

- differentiability, 254, 257
- discriminant, 4
- disjunction, 69, 71
- distributivity, 44, 308
- divisibility, 164
- divisible by, 17
- Division Algorithm, 164, 165
- domain, 36

- elementary statement, 73
- end of proof symbol \square , 5
- equivalence, 69, 71
 - class, 191
 - relation, *see* relation
- equivalent vectors, 331
- Erdős–Szekeres Theorem, 230
- Euclid’s Lemma, 175
- Euclidean Algorithm, 168–171
- Euler’s Identity, 295
- even number, 18

- factorial, 107
- field
 - axioms, 25, 43
 - of complex numbers, 270
 - of rational functions, 47
 - ordered, 62
 - with two elements, 47
- function, 25, 35, 36
 - as a relation, 186
 - bijjective, *see* bijection
 - bounded, 58, 85
 - composition, 135
 - decreasing, 133
 - graph of, 38
 - image of, 36
 - increasing, 133
 - injective, *see* injection
 - inverse, 129
 - monotone, 133
 - not properly defined, 52
 - range of, 36
 - surjective, *see* surjection
 - unbounded, 85
 - well-defined, 54
- Fundamental Theorem of Algebra, 292
- Fundamental Theorem of Arithmetic, 175, 177

- GCD, *see* greatest common divisor
- geometric mean, 9
- graph (of a function), *see* function
- greatest common divisor, 168

- identity
 - additive, 44
 - multiplicative, 44
- if and only if, 6
- image
 - of a function, *see* function
 - of a set, 41
 - of an element, 36
- implication, 69, 71, 74
- Inclusion-Exclusion Principle, 221, 225
- induction, *see* mathematical induction
- inequalities, 6
- infinite intersections, 49
- infinite unions, 49
- injection, 128
- integer numbers, 17, 26
- Intermediate Value Theorem, 257
- interval
 - closed, 29
 - notation, 29
 - open, 29
- inverse
 - additive, 45
 - multiplicative, 45
- irrational numbers, 19
- isomorphism, 325, 330

- L-tiling, 111
- LCM, *see* least common multiple
- least common multiple, 181
- limit
 - at infinity, 262
 - infinite, 262
 - infinite (for sequences), 261
 - of a function, 242, 248
 - of a sequence, 234, 236
 - one-sided, 250, 251

- linear algebra, 306
- linear map, 325, 326
 - image of, 327
 - kernel (null space) of, 327
- logic symbols, 70
- logical equivalence, 77
- logically equivalent statements, 79
 - with quantifiers, 82
- mathematical induction, 99
 - strong, 115
 - the principle of, 99, 100
 - variations, 109
- natural numbers, 16, 26
- negation, 69, 71, 82
- neighbourhood, 243
 - punctured, 243
- odd number, 18
- order axioms, 49
- Parallelogram Identity, 296
- Parallelogram Law, 313
- partition, 195
- Pascal's Identity, 216
- Pascal's Triangle, 215, 218
- permutation, 207, 209, 210, 228
- pi notation, *see* product notation
- Pigeonhole Principle, 219
 - generalized, 220
- positive integers, 16
- power set, *see* set
- predicate, 68
- prime number, 18, 164
 - infinitely many, 91
- product notation, 106
- proof
 - by contradiction, 88
 - by contrapositive, 88
 - direct, 87
 - strategies, 86
- proposition, 67
- quadratic equation, 3
 - complex, 280, 283
- quadratic formula, 3
- quantifier, 68, 76
 - existential, 71
 - universal, 71
- quotient set, 195
- \mathbb{R}^2 , 32
- \mathbb{R}^n , 306
- range, *see* function
- rational numbers, 18, 26
- real number system, 19, 49
- real numbers, 4, 19
- recursion, 112
- reflexivity, *see* relation
- relation, 184, 185
 - binary, 185
 - equality, 190
 - equivalence, 187
 - reflexive, 187
 - symmetric, 187
 - transitive, 187
- relatively prime, 168
- remainder, 165
- rough work, 11
- Rule of Product, 208
- Rule of Sum, 208, 221
- scalar multiplication, 307, 308, 314
- Schröder–Bernstein Theorem, 151, 152, 160
- selection, 207, 211
 - with repetitions, 212
- set, 25
 - complement, 30
 - countable, 147
 - element of, 25
 - empty, 26
 - identities, 34
 - null, 26
 - operations, 29
 - power, 149
 - uncountable, 147
 - universal, 30
- set-builder notation, 26
- sets
 - Cartesian product of, 30, 34
 - difference of, 30

- sets (cont.)
 - disjoint, 30
 - intersection of, 30
 - union of, 30
- sigma notation, *see* summation notation
- statement, 67
 - compound, 70
 - vacuously true, 86
- stick and stones, 212
- subfield, 62
- subset, 26
 - of a finite set, 104
- summation notation, 106
- surjection, 128
- symmetry, *see* relation
- tip-to-tail rule, 314
- transitivity, *see* relation
- Triangle Inequality, 9, 13, 15
 - for complex numbers, 276, 278
 - general, 100, 103
 - reverse, 22
- truth table, 73, 77
 - elementary, 77
- truth value, 73
- types of numbers, 16
- variation, 209
- vector
 - addition (in \mathbb{R}^n), 307, 308
 - addition (of geometric vectors), 313
 - algebraic, 306
 - coordinates, 306
 - geometric, 309
 - length, 331
 - space, 317
 - subspace, 321, 322
- vector space
 - sum, 325
- Venn diagram, 30
- Vieta's Formulas, 20
- Weierstrass' First Theorem, 257
- Weierstrass' Second Theorem, 257