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Excerpt
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PART I

The Republic

CHAPTER 1

Plato on why mathematics is good for the soul

1 The question

Anyone who has read Plato's *Republic* knows it has a lot to say about mathematics. But why? I shall not be satisfied with the answer that the future rulers of the ideal city are to be educated in mathematics, so Plato is bound to give some space to the subject. I want to know why the rulers are to be educated in mathematics. More pointedly, why are they required to study so much mathematics, for so long?

They start in infancy, learning through play (536d–537a). At 18 they take a break for two years' military training. But then they have another ten years of mathematics to occupy them between the ages of 20 and 30 (537bd). And we are not talking baby maths: in the case of stereometry (solid as opposed to plane geometry), Plato has Socrates make plans for it to develop more energetically in the future (528bd), because it only came into existence (thanks especially to Theaetetus) well after the dramatic date of the discussion in the *Republic*. Those ten years will take the Guards into the most advanced mathematical thinking of the day. At the same time they are supposed to work towards a systematic, unified understanding of subjects previously learned in no particular order (χύδην). They will gather them together to form a synoptic view of all the mathematical disciplines 'in their kinship with each other and with the nature of what is' (537c). I shall come back to this enigmatic statement later. Call it, for the time being, Enigma A. ||

The extent of mathematical training these people are to undergo is astounding. They are not preparing to be professional mathematicians; nothing is said about their making creative contributions to the subject. Their ten years will take them to the synoptic view, but then they switch to dialectic and philosophy. They are being educated for a life of philosophy and government. How, we may ask, will knowing how to construct an

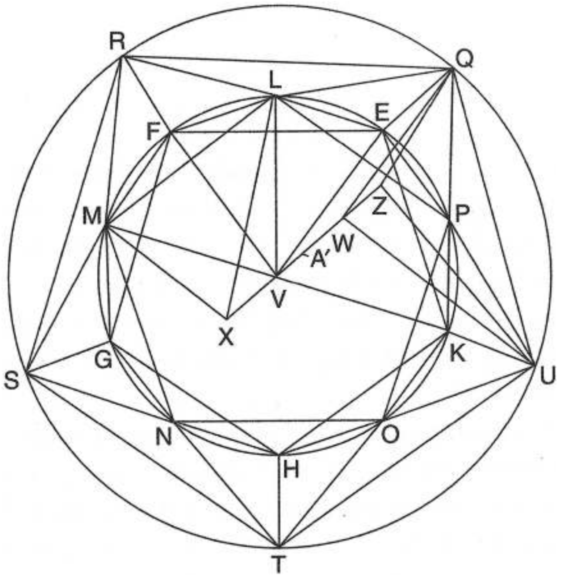

Figure 1.1 Construction of an icosahedron.

icosahedron (Figure 1.1) help them when it comes to regulating the ideal market or understanding the Platonic Theory of Forms?

The question is reminiscent of debates in the not so distant past about the value of a classical education. Why should the study of Greek and Latin syntax be advocated, as once it was, as the ideal preparation for entering the Civil Service or the world of business? No doubt, any rigorous discipline helps train the mind and imparts 'transferable skills'. But that is no reason to make Latin and Greek compulsory when other disciplines claim to provide equal rigour, e.g. mathematics. Conversely, readers of the *Republic* are entitled to put the question to Plato: why so much mathematics, rather than something else? ||

All too few scholars put this question, and when they do, they tend to answer by stressing the way mathematics trains the mind. Plato 'is proposing a curriculum for mental discipline and the development of abstract thought'; he believes no one can become 'a moral hero or saint' without 'discipline in sheer hard thinking'; he advocates mathematics 'not simply because it involves turning away from sense perception but because it is constructive reasoning pursued without reference to immediate instrumental usefulness'.[1] Like the dry-as-dust classicists for whom the value of learning Greek had nothing to do with the value of reading Plato or Homer, this type of answer implies that the *content* of the mathematical curriculum is irrelevant to its goal. At best, if the chief point of mathematics is to encourage the mind in abstract reasoning, the curriculum may help rulers to reason abstractly about non-mathematical problems in ethics and politics.

One ancient writer who did think that mind-training is the point was Plato's arch-rival, the rhetorician Isocrates (*Antidosis* 261–9, *Panathenaicus* 26–8). Speaking of the educational value of mathematics and dialectic, he said it is not the knowledge you gain that is beneficial, but the process of acquiring it, which demands hard thought and precision.[2] From this he concluded, quite reasonably, that young men should not spend too much time on mathematics and dialectic. Having sharpened up their minds, they should turn to more important subjects like public speaking and government. Isocrates was not trying to elucidate Plato's thought. He was sketching a commonsensical *alternative* role for mathematics and || dialectic, to counteract the excessive claims coming from the Academy. Mathematics and dialectic would hone young minds for an education in rhetoric.

Isocrates presents himself as taking a conciliatory approach on a controversial issue. Most people, he says, think that mathematics is quite useless for the important affairs of life, even harmful. No one would say that now, because we live in a world which in one way or another has been transformed by mathematics. No one now reads Sir William Hamilton on the bad effects of learning mathematics, so no one needs John Stuart Mill's vigorous and moving riposte.[3] In those days, however, a sophist like Protagoras could openly boast about saving his pupils the bother of learning the 'quadrivium' (arithmetic, geometry, astronomy, and harmonics), which his rival Hippias insisted on teaching; instead of spoiling his pupils' minds with mathematics, Protagoras would proceed at once to

[1] Quoted from, respectively, Shorey 1933: 236; A. E. Taylor 1926: 283; Irwin 1995: 301.

[2] Quintilian, *Institutio oratoria* I 10.34, describes this as the common view (*vulgaris opinio*) of the educational value of mathematics, and goes on to assemble more substantive (but still instrumental) reasons why an orator needs a mathematical training. Galen, περὶ ψυχῆς ἁμαρτημάτων 49.24–50.l Marquardt, mentions a variety of disciplines by which the soul is sharpened (θήγεται) so that it will judge well on practical issues of good and bad: logic, geometry, arithmetic, calculation (λογιστική), architecture, and astronomy. If architecture (as a form of technical drawing), why not engineering? And what could beat librarianship for encouraging a calm, orderly mind?

[3] Sir William Hamilton, 'On the Study of Mathematics, as an Exercise of Mind', Hamilton 1852: 257–327; MCW IX, ch. 27.

what they really wanted to learn, the skills needed to do well in private and public life (Plato, *Protagoras* 318de). At a more philosophical level, Aristippus of Cyrene, who like Plato had been a pupil of Socrates, could lambast mathematics because it teaches nothing about good and bad (Aristotle, *Metaphysics* B.2, 996a32–b1). Xenophon's Socrates contradicts Plato's by setting narrowly practical limits to the mathematics required for a good education: enough geometry to measure land, enough astronomy to choose the right season for a journey. Anything more complicated, he says, is a waste of time and effort, while it is impious for astronomers to try to understand how God contrives the phenomena of the heavens (*Memorabilia* IV.7.1–8).

These ancient controversies show that the task of persuasion Plato set himself was still harder then than it would be today. Even Isocrates' mind-sharpening recommendation could not be taken for granted.

A very different account of the mind-sharpening value of mathematics can be found in a later Platonist (uncertain date AD) || called Alcinous, who says that mathematics provides the precision needed to focus on real beings, meaning abstract, non-sensible beings (*Didaskalikos* 161.10–13ff.).[4] As we shall see, mathematical objects can only be grasped through precise definition, not otherwise, so there is good sense in the idea that precision is the essential epistemic route to a new realm of beings.[5] In that spirit, more enlightened classicists promote Greek and Latin as a means of access to a whole new realm of poetry and prose which you cannot fully appreciate in translation.

This seems to me a more satisfactory version of the mind-sharpening view than we find in Isocrates, who thinks of mathematics as providing a content-neutral ability you can apply to any field. But I shall argue that Alcinous still does not go far enough. My comparison would be with a classicist who dared claim that embodied in the great works of antiquity is an important part of the truth about reality and the moral life.[6]

The goal of the mathematical curriculum is repeatedly said to be knowledge of the Good (526de, 530e, 531c, 532c). That ten-year immersion in mathematics is the propaedeutic prelude (531d, 536d) to five years' concentrated training in dialectical discussion (539de), which will eventually lead the students to knowledge of the Good. I say 'eventually', because

4 Alcinous' phrase is θήγουσα τὴν ψυχήν, as in Galen (n. 2). The Latin equivalent is *acuere*: Quintilian, *Institutio oratoria* I.10.34, Cicero, *De Republica* I.30.
5 For a comparable approach today, see Annas 1981: 238–9, 50–1, 72–3.
6 In this approach my closest ally is Gosling 1973, ch. 7, but see also, briefly, Cooper 1977: 155, reprinted in Cooper 1999: 144, and Lennox 1985b: 215–18.

at the age of 35 they break off for fifteen years' practical experience in a variety of military and administrative offices (539e–540a). Only when they reach 50 do they resume dialectic for the final ascent to see the Good, the *telos* for which their entire education has been designed (540ab). Knowledge of the Good is obviously relevant to government and to philosophy. So || my question can be put like this: Is the study of mathematics merely instrumental to knowledge of the Good, in Plato's view, or is the content of mathematics a constitutive part of ethical understanding? I shall argue for the latter.[7]

2 Outline of the answer

To launch this idea, and to help make it, if not palatable, at least more intelligible than it is likely to be at first hearing, I shall take a modern foil – a tough-minded logical empiricist of the twentieth century, whose argument I find both strikingly reminiscent of Plato's *Republic* and revealingly different:

> We walk through the world as the spectator walks through a great factory: he does not see the details of machines and working operations, or the comprehensive connections between the different departments which determine the working processes on a large scale. He sees only the features which are of a scale commensurable with his observational capacities: machines, workingmen, motor trucks, offices. In the same way, we see the world in the scale of our sense capacities: we see houses, trees, men, tools, tables, solids, liquids, waves, fields, woods, and the whole covered by the vault of the heavens. This perspective, however, is not only one-sided; it is false, in a certain sense. Even . . . the things which we believe we see as they are, are objectively of shapes other than we see them. We see the polished surface of our table as a smooth plane; but we know that it is a network of atoms with interstices much larger than the mass particles, and the microscope already shows not the atoms but the fact that the apparent smoothness is not better than the 'smoothness' of the peel of a shriveled apple. We see the iron stove before us as a model of rigidity, solidity, immovability; but we know that its particles perform a violent dance, and that it resembles a swarm of dancing gnats more than the picture of solidity we attribute to it. We see the moon as a silvery disk in the celestial vault, but we know it is || an enormous ball suspended in open space. We hear the voice coming from the mouth of a

[7] This will involve revisiting a number of themes I discussed in Burnyeat 1987, republished as *EAMP* Vol. II, ch. 7, 145–72. But here they will receive a more expansive treatment, with fewer references to the scholarly literature than was appropriate to the earlier Symposium. Naturally, I cannot promise to be entirely consistent now with what I wrote then.

> singing girl as a soft and continuous tone, but we know that this sound is composed of hundreds of impacts a second bombarding our ears like a machine gun. The [objects] as we see them have as much similarity to the objects as they are as the little man with the caftan seen in the moor [at dusk from afar] has to the juniper bush [it turns out to be], or as the lion seen in the cinema has to the dark and bright spots on the screen. We do not see the things . . . as they are but in a distorted form; we see a *substitute world* – not the world as it is, objectively speaking.

So wrote Hans Reichenbach in 1938.[8] The idea he formulates of the world as it is objectively speaking is the idea of what the world is discovered to be when one filters out the cognitive effects of our human perspective. More fully, it is the idea of the world described in a way that takes account of all the aspects we miss from our usual perspective, so as to explain why we experience it as we do: the moon is both a silvery disk and an enormous ball far away, and it is the one because it is the other. This idea, I claim, received its first full-scale formulation and defence in the central Books of Plato's *Republic*. Reichenbach's cinema is a twentieth-century version of Plato's famous simile of the cave. Plato is the better poet, but his philosophy is no less tough-minded. Both cinema and cave make us look at our ordinary experience of the world from the outside, as it were, to see how inadequate it is by comparison with the view we would have from the standpoint of a scientific account of the world as it is objectively speaking. The cinema analogy, like the Cave, expresses the idea that human experience is just a particular, parochial perspective which we must transcend in order to achieve a full, accurate, and properly explanatory view of things.

So much for the similarity. But of course there are also differences. Reichenbach can put across his version of the idea in a couple of pages, because his readers grew up in an age already familiar with the contrast between the world as humans experience || it and the world as science explains it. In Plato's time the idea was a novelty, harder to get across. Moreover, Plato was addressing a wider readership than a technical book of modern philosophy can hope to reach. His readers have further to travel from where they start to where he wants them to end up. They need the imagery and the panoply of persuasive devices that enliven the long argument of *Republic* Books v–vii.

[8] Reichenbach 1938: 219–20, omitting three occurrences of his technical term 'concreta'; the example of the little man with the caftan was introduced at p. 198. In his Preface Reichenbach aligns himself with philosophical movements which share 'a strict disavowal of the metaphor language of metaphysics'!

Another difference is that Reichenbach can rest on the *authority* that science enjoys in the modern world. In Plato's day no system of thought or explanation had such authority. Everything was contested, every scheme of explanation had to compete with rivals. Modern logic is a further resource that Reichenbach can take for granted. In Plato's day logic was not yet invented, let alone established. Methods of reasoning and analysis were as contested as the content they were applied to.

But the really big difference between Reichenbach's and Plato's version of the idea of the world as it is objectively speaking is the following. For Reichenbach in the twentieth century the world as it is objectively speaking is the world as described by modern science, above all mathematical physics, and in that description there is no room for values. The world as it is objectively speaking, seen from the standpoint of our most favoured science, is a 'disenchanted' world without goodness in it. For Plato, by contrast, the most favoured science – in his case, mathematics – is precisely what enables us to understand goodness. The mathematical sciences are the ones that tell us how things are objectively speaking, and they are themselves sciences of value. Or so I shall argue. If I am right, understanding the varieties of goodness is for Plato a large part of what it means to understand the world as it is objectively speaking, through mathematics. Plato, like Aristotle and the Stoics after him, really did believe there is value in the world as it is objectively speaking, that values are part of what modern philosophers like to call 'the furniture of the world'.

This is not the place or the time to consider how and why the world became 'disenchanted'. Let it be enough that an understanding of impersonal, objective goodness is for Plato the climax and *telos* of an education in mathematics. It is this concept of || impersonal, objective goodness that links the epistemology and metaphysics of the *Republic* to its politics. Plato's vision of the world as it is objectively speaking is the basis, as Reichenbach's could never be, for a political project of the most radical kind. The moral of the Cave is that Utopia can be founded on the rulers' knowledge of the world as it is objectively speaking, because that includes the Good and the whole realm of value.

3 By-products

It is relatively easy to prove the negative point that Socrates in the *Republic* does not recommend mathematics solely for its mind-training, instrumental value. He says so himself.

We may start with arithmetic. Socrates gives three reasons why this is a 'must' (ἀναγκαῖον – 526a8) for the further education of future rulers. His chief reason, expounded at length, is that arithmetic forces the soul towards an understanding of what numbers are in themselves, and thereby focuses thought on a realm of unqualified truth and being (526b, summing up the result of 524d–526b). More about that later. Then he adds two further reasons, each stated briefly. First, arithmetic makes you quicker at other studies, all of which involve number in some way (526b with 522c); this sounds like what we call transferable skills. Second, the subject is extremely demanding to learn and practise (526c); as such, it is a good test of intellectual and moral calibre (cf. 503ce, 535a–537d).

Thus far the relative ranking of intrinsic and instrumental benefits is left implicit. The next section, on (plane) geometry, should leave an attentive reader in no doubt where Plato's priorities lie. Having recommended that geometry be studied for the sake of knowing what everlastingly is, not for the sake of action in the here and now (527ab),[9] Socrates acknowledges that, besides its capacity to drag the soul upwards towards truth, geometry has certain by-products (πάρεργα) which are, he says, 'not small', namely, 'its uses in war, which you mentioned just now, and besides, for the || better reception of all studies we know there will be an immeasurable difference between a student who has been imbued with geometry and one who has not' (527c).[10] The term 'by-products' should be decisive. Both the practical application of geometry in war (e.g. for troop formation and the laying out of camp sites – 526d) and transferable skills are relegated to second rank in comparison to pure theoretical knowledge. Plato would hardly write in such terms if he valued geometry for content-neutral skills that the Guards can later apply when ruling or trying to understand the Good. This conclusion is reinforced when we see that the passage belongs to a sequence of episodes which climax in a strong denunciation of any demand for the curriculum to be determined by its practical pay-off.

At the start of the discussion Socrates made a point of saying that any studies chosen for the curriculum must not be useless (note the double negative) for warriors. This is because he and Glaucon are planning the further education of people who have been trained so far to be 'athletes in

9 So too arithmetic should be studied for the sake of knowledge, not trade (525d).
10 Translations from the *Republic* are my own, but I always start from Shorey's Loeb edition (1935, 1937), so his phrases are interwoven with mine. For passages dealing with music theory, I have borrowed freely from the excellent rendering (with useful explanatory notes) given by Andrew Barker in Barker 1984–9. It will become clear how much, as a beginner in mathematical harmonics, I owe to Barker's work.

war' (521d). Arithmetic satisfies that condition, he argues, because a warrior must be able to count and calculate (522e). True, but that is hardly adequate justification for ten years' immersion in number theory. Notice, however, that the justification is introduced by a joke: how ridiculous Agamemnon is made to look in the tragedies which retail the myth that Palamedes was the discoverer of number, the one who marshalled the troops at Troy and counted the ships. As if until then Agamemnon did not even know how many feet he had (522d)! Glaucon agrees. The ability to count and calculate is indeed a 'must' for a warrior, if he is to understand anything about marshalling troops – or rather, Glaucon adds, if he is to be a human being (522e). This last is the give-away. Plato is not serious about || justifying the study of arithmetic on grounds of its practical utility. His real position becomes clear later (525bc): while it is true that a warrior needs the arithmetical competence to marshal troops in the world of becoming, a *philosopher* needs to study arithmetic for the quite different reason that it turns the soul away from the world where battles are fought. The Guards will continue to be warriors as well as philosophers, but it is their philosophical education that is top of the agenda now.[11]

Glaucon is slow to grasp the point. When the discussion turns to (plane) geometry, it is he who enthuses about the importance of geometry for laying out camp sites, occupying territory, closing up or deploying an army, and manoeuvring in battle or on the march (526d). Socrates drily responds that you do not need much geometry (or calculation) for things like that. What we should be thinking about, he says, is whether geometry – geometry at an advanced level[12] – will help one come to know the Good (526de).

Plato did not write these exchanges just to have some fun at his brother's expense. He is preparing a surprise for his readers. The surprise comes when we reach astronomy. Glaucon duly commends the study on the grounds that generals, like sailors and farmers, need to be good at telling the seasons (527d). (Invading armies should beware of Russia in the winter

[11] The distinction of roles (warrior *vs* philosopher) provides the context for the claim at 525c that arithmetic should be studied 'both for the sake of war and to attain ease in turning the soul itself from the world of becoming to truth and reality' (525c4–7), about which Annas unfairly remarks, 'This utterly grotesque statement may sum up quite well the philosophy behind a lot of NATO research funding' (Annas 1981: 275). It would be more apt to wonder how the distinction of roles squares with the 'one man–one job' principle on which the ideal city was founded in Book II.

[12] So too arithmetic should be taken to an advanced level (525c: μὴ ἰδιωτικῶς).