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PROOFS AND MODELS IN PHILOSOPHICAL LOGIC

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Proofs and Models in Philosophical Logic

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Abstract: This Element is an introduction to recent work on proofs and models in philosophical logic, with a focus on the semantic paradoxes and the sorites paradox. It introduces and motivates different proof systems and different kinds of models for a range of logics, including classical logic, intuitionistic logic, a range of three- and four-valued logics, and substructural logics. It also compares and contrasts the different approaches to substructural treatments of the paradox, showing how the structural rules of contraction, cut and identity feature in paradoxical derivations. It then introduces model theoretic treatments of the paradoxes, including a simple fixed-point model construction that generates three-valued models for theories of truth, which can provide models for a range of different non-classical logics. The Element closes with a discussion of the relationship between proofs and models, arguing that both have their place in philosophers' and logicians' toolkits.

Keywords: proofs, models, logic, semantics, paradox

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