General Relativity: The Essentials

In this short book, renowned theoretical physicist and author Carlo Rovelli gives a straightforward introduction to Einstein's general relativity, our current theory of gravitation. Focusing on conceptual clarity, he derives all the basic results in the simplest way, taking care to explain the physical, philosophical, and mathematical ideas at the heart of 'the most beautiful of all scientific theories'. Some of the main applications of general relativity also are explored, for example, black holes, gravitational waves, and cosmology, and the book concludes with a brief introduction to quantum gravity. Written by an author well known for the clarity of his presentation of scientific ideas, this concise book will appeal to university students looking to improve their understanding of the principal concepts, as well as to science-literate readers who are curious about the real theory of general relativity, at a level beyond a popular science treatment.

CARLO ROVELLI is Director of the Quantum Gravity group at the Centre de Physique Théorique of Aix-Marseille University; he also holds positions at the University of Western Ontario and the Perimeter Institute in Canada. Among his academic contributions in theoretical physics, he is best known as one of the formulators of Loop Quantum Gravity. He has written two monographs for Cambridge University Press, *Quantum Gravity* (2004) and (with Francesca Vidotto) *Covariant Loop Quantum Gravity* (2014). He is also the author of several international bestsellers in popular science such as *Seven Brief Lessons on Physics* (2016) and *The Order of Time* (2017).
General Relativity: The Essentials

CARLO ROVELLI
Université d'Aix-Marseille
## Contents

Preface  

<table>
<thead>
<tr>
<th>PART I  BASES</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>What Is General Relativity?</td>
<td>1</td>
</tr>
<tr>
<td>1 Physics: A Field Theory for Gravity</td>
<td>3</td>
</tr>
<tr>
<td>1.1 Special Relativity</td>
<td>3</td>
</tr>
<tr>
<td>1.2 Fields</td>
<td>7</td>
</tr>
<tr>
<td>2 Philosophy: What Are Space and Time?</td>
<td>11</td>
</tr>
<tr>
<td>2.1 Relative versus Newtonian Space and Time</td>
<td>11</td>
</tr>
<tr>
<td>2.2 Einstein’s Idea: Newtonian Space and Time Are a Physical Field</td>
<td>13</td>
</tr>
<tr>
<td>2.3 Einstein’s Hint: Acceleration with Respect to What?</td>
<td>15</td>
</tr>
<tr>
<td>3 Mathematics: Curved Spaces</td>
<td>19</td>
</tr>
<tr>
<td>3.1 Curved Surfaces</td>
<td>19</td>
</tr>
<tr>
<td>3.2 Riemannian Geometry</td>
<td>33</td>
</tr>
<tr>
<td>3.3 Geometry</td>
<td>50</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>PART II  THE THEORY</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>4 Basic Equations</td>
<td>59</td>
</tr>
<tr>
<td>4.1 Gravitational Field</td>
<td>59</td>
</tr>
<tr>
<td>4.2 Effects of Gravity</td>
<td>60</td>
</tr>
<tr>
<td>4.3 Field Equations</td>
<td>61</td>
</tr>
<tr>
<td>4.4 Source in the Field Equation</td>
<td>62</td>
</tr>
<tr>
<td>4.5 Vacuum Equations</td>
<td>63</td>
</tr>
</tbody>
</table>
CONTENTS

5 Action 65

6 Symmetries and Interpretation 68
   6.1 Time and Energy 71

PART III APPLICATIONS

7 Newtonian Limit 77
   7.1 The Metric in the Newtonian Limit 77
   7.2 Newton's Force 78
   7.3 General Relativistic Time Dilatation 79

8 Gravitational Waves 83
   8.1 Effect on Matter 87
   8.2 Production and Detection 90

9 Cosmology 95
   9.1 The Large-Scale Geometry of the Universe 98
   9.2 Basic Cosmological Models 102

10 The Field of a Mass 106
   10.1 Schwarzschild Metric 106
   10.2 The Kepler Problem 108
   10.3 Deflection of Light by the Sun 114
   10.4 Near Horizon Orbits 116
   10.5 Cosmological Force 119
   10.6 Kerr–Newman Metric and Frame Dragging 120

11 Black Holes 123
   11.1 At the Horizon 124
   11.2 Inside the Black Hole 129
   11.3 White Holes 132
## CONTENTS

12 Elements of Quantum Gravity 140
12.1 Empirical and Theoretical Basis of Quantum Gravity 141
12.2 Discreteness: Quanta of Space 143
12.3 Superposition of Geometries 148
12.4 Transitions: Black-to-White-Hole Tunnelling and Big Bounce 152
12.5 Conclusion: The Disappearance of Spacetime 155

Further Reading 157
Index 161
Preface

There are absolute masterpieces, that move us intensely, Mozart’s Requiem, the Odyssey, the Sistine Chapel, King Lear... To grasp their splendour may require an apprenticeship. But the prize is pure beauty. And not only: also the opening of a new look at the world to our eyes.

General relativity, the jewel of Albert Einstein, is one of them.

This short book offers a compact introduction to general relativity, its conceptual structure, and its basic results.

The focus is on the ideas, without extensive details. The main results are derived in their simplest form, without lengthy mathematical manipulations. Some original considerations are included, and some topics are discussed from a perspective not easily found elsewhere. A final chapter introduces elementary ideas on quantum gravity. The book can be used to learn key ideas and results of general relativity without the ambition of becoming fully expert on its vast ramifications.

It can also be used as a complement to the numerous extensive manuals, offering additional conceptual clarity. It presents general relativity in the way I understand it today, which I think is the best perspective to address its quantum aspects.

---

1 A good and motivated student looks at many books on the same topic. Two classics I still use as references are Bob Wald's *General Relativity*, which is mathematically oriented, emphasises the geometrical perspective, and has a lot of advanced material, and Steven Weinberg's *Gravitation and Cosmology*, which de-emphasises geometry. Modern textbooks are Sean Carroll's *Spacetime and Geometry* and Lewis Ryder’s *Introduction to General Relativity*; these are vastly more comprehensive than the simple introduction given here. A good mathematically oriented text is Yvonne Choquet-Bruhat’s *Introduction to General Relativity, Black Holes, and Cosmology*. In French, a nice introduction is *Relativité générale: Cours et exercices corrigés* by Aurélien Barrau. This is to mention only the few I know best.
Simple mathematical steps between equations are skipped and indicated with the text ‘[do it!]’. The reader can trust the author, as physicists often do when reading maths; or they can work out the steps and acquire technical competence. A student that does so will end up with good hands-on experience of the technology of relativity. If you like doing exercises, there are several books of exercises in general relativity.\footnote{For instance, Thomas A. Moore’s \textit{A General Relativity Workbook}.}

The texts in small characters are somewhat marginal with respect to the main topics.

Thanks to Pietropaolo Frisoni for the many corrections and for the translation of the text into Italian. Thanks also to Aymeric Derville for spotting many typos and mistakes in the first draft. I am sure there are more: if you find them, I will be grateful if you can point them out to me.

The recent Nobel Prizes for general relativistic physics (gravitational waves 2017, cosmology 2019, black holes 2020) are a testament to the current vitality and fecundity of this extraordinary theory, Einstein’s jewel. Here I try to bring to light the shining beauty and simplicity of the ideas on which it is based.