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978-1-009-66585-2 — Effective Results and Methods for Diophantine Equations over Finitely Generated Domains
Jan-Hendrik Evertse , Kálmán Győry
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