

Fundamentals of Transport Processes

The study of transport phenomena is an essential part of chemical engineering, as well as other disciplines concerned with material transformations such as biomedical engineering, mechanical engineering and materials engineering. Material transformations require the motion of constituents relative to each other, the transfer of heat across materials and fluid flow.

This lucid textbook introduces the student to the fundamentals and applications of transport phenomena in a single volume, and explains how the outcomes of transformation processes depend on fluid flow and heat/mass transfer. It demonstrates the progression from physical concepts to the mathematical formulation, followed by the solution techniques for predicting outcomes in industrial applications. The ordering of the topics, gradual build-up of complexity and easy to read language make it a vital resource for anyone looking for an introduction to the domain. It also provides a foundation for advanced courses in fluid mechanics, multiphase flows and turbulence. The author explains the book in a series of video lectures in the supplements package.

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V. Kumaran



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Dedicated to the memory of my father,
whose belief in my abilities was the ‘driving force’
that ‘transported’ me to where I am.

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Preface

An anecdote about Prof. P. K. Kelkar, founding director of IIT Kanpur and former director of IIT Bombay, was narrated to me by Prof. M. S. Ananth, my teacher and former director of IIT Madras. A distraught young assistant professor at IIT Kanpur approached the director and complained that ‘the syllabus for the course is too long, and I am will not be able to cover everything’. Prof. Kelkar replied, ‘You do not have to cover everything, you should try to uncover a few things.’ In this book, my objective is to uncover a few things regarding transport processes.

The classic books on transport processes, notably the standard text *Transport Phenomena* by Bird, Stewart and Lightfoot written about 60 years ago, provided a comprehensive overview of the subject organised into different subject areas. At that time, engineers were required to do design calculations and modeling for different unit operations, and for the sequencing of these operations in process design. This required expertise in laboratory and pilot scale experiments on unit operations and scaling up of these operations using correlations. Proficiency in developing, understanding and using design handbooks and correlations was also needed. In this context, the study of transport processes at the microscopic level, and its implications for design for unit operations, was a pioneering advance that has since become an essential part of the chemical engineering curriculum.

In the last half century, sophisticated computational tools have been developed for detailed flow modeling within unit operations, and for the selection and concatenation of unit operations for achieving the required material transformations. The ease of search for information and data today was inconceivable half a century ago. Routine calculations have been automated, and there is little need for routine tasks such as unit conversion, graphical construction and interpreting engineering tables. There is now a greater need for understanding physical phenomena and processes and their mathematical description.

Using a rigorous understanding of transport processes, an engineer usually contributes to process design in one of two ways. The first is the development and enhancement of models and computational tools for modeling of flows

and transformations in unit operations; these result in higher resolution, better representation of the essential physics and inclusion of new phenomena. The second is the use of these tools for design of unit operations, and the coupling between them. In the present context, the objective of this text is to assist the student in internalising some common conceptual frameworks for transport processes.

- (1) At the level of unit operations, the use of dimensional analysis is enhanced to provide a physical understanding of the dimensionless groups for common internal and external flows in chapters 1–2. Justification is provided for the forms of the correlations in different parameter regimes.
- (2) The interplay between convective and diffusive transport is one of the central ideas. This is emphasised in chapter 2, and the molecular origins of diffusion are discussed in chapter 3. Approximate methods for estimating diffusion coefficients in gases and liquids are developed.
- (3) The progression from the balance condition for a differential volume, the inclusion of the constitutive relation for transport across surfaces, to the derivation of the partial differential equation for the densities of mass/momentum/energy is shown in chapter 4 for a Cartesian co-ordinate system and chapter 5 for curvilinear co-ordinate systems. This demonstrates how the mathematical formulation arises from the physical description.
- (4) The procedure for deriving conservation equations for a general co-ordinate system is the subject of chapter 7. A compact representation in terms of vector differential operators is obtained, and the operators are determined for different co-ordinate systems. This enables a student to comprehend the meaning of the vector notation, and the form of the vector operators in different co-ordinate systems.
- (5) The use of physical insight in reducing partial differential equations to one or more ordinary differential equations is demonstrated in chapters 4 and 5 for unidirectional transport. Two procedures are emphasised, similarity transforms and separation of variables.
- (6) The similarity transform procedure is then developed into the concept of a boundary layer in chapter 9 for forced convection, and chapter 10 for natural convection. This is an important conceptual basis for transport under strong convection, and it provides a means for understanding the form of the correlations for laminar flows.
- (7) The separation of variables procedure is used for solving the diffusion equation in Cartesian and spherical co-ordinates in chapter 8. This is further developed

to the spherical harmonic analysis in a spherical co-ordinate system, and the resulting fields are interpreted in terms of sources and sinks.

- (8) The role of pressure in momentum transport is the subject of chapter 6. After examining potential flows, the laminar and turbulent flows in a pipe are discussed in detail. The discussion of a turbulent flow is more detailed than that in most courses on the same subject, because it is important for a student to have a physical picture of the structure of a turbulent flow. The Navier–Stokes equations for a Newtonian fluid are presented without derivation in chapter 7, along with reasonably advanced treatments of the stress and rate of deformation tensor in a fluid.

Some important topics that do not fit into the set of common frameworks listed above are not included in this study. Multicomponent diffusion is discussed at a very basic level in chapter 4, only for binary mixtures. The derivation of the diffusion flux using non-equilibrium thermodynamics is a specialised topic which is not discussed here. Radiation heat transfer is also not examined here, since it is not possible to do justice to this without a basic coverage of electromagnetic waves.

Computational methods are also not covered in this course. There have been significant advances in computational methods in the last few decades, and one or more specialised courses are required to cover this area to a reasonable extent.

Some of the content in the book requires only a rudimentary knowledge of mathematics, while there is more advanced material requiring calculus, special functions and solution procedures for ordinary differential equations. All of the course material is suitable for students who have taken undergraduate courses in calculus, linear algebra and ordinary differential equations. Where necessary, additional background for the mathematical formulation has been provided in the appendices.

For students without adequate exposure to mathematical methods, an elementary course could be designed by selecting topics from chapters 1, 2, 3, 4.1–4.3, 4.8, 5.1.1–5.1.2, 5.2.1–5.2.3 and 6.1–6.3. The remaining chapters and sections could constitute an advanced course which requires proficiency in advanced mathematical techniques at the undergraduate level.

The important concepts in each section are emphasised in the summary at the end of the section. The important equations in the section are enclosed in boxes, and these are also highlighted in the summary section. There are numerical examples throughout the text, which enable the students to acquire a feel for the numbers and magnitudes involved in practical situations. An adequate number of exercise

problems are provided at the end of each chapter, which can be solved using procedures discussed in the respective chapter.

At the end of the course, it is hoped that the conceptual understanding provided here enables the prospective engineer to develop a ‘feel’ for the phenomena involved when faced with a new problem, assess the dominant forces and the flow regime from dimensionless groups, and carry out quick estimates of the magnitudes of the transport rates which can be used to evaluate feasibility. These estimates can also be used to check the consistency of solutions from more rigorous analytical or numerical procedures. When using computational tools, it is important to know the operating flow regimes so that the appropriate models can be used, and to validate the consistency of the numerical scheme with known analytical solutions in limiting parameter regimes. An understanding of the meaning of the conservation equations in vector notation, and their representation as partial differential equations in different co-ordinate systems, is essential for those using computational tools to determine the field variables within unit operations. The book is also a useful starting point for prospective researchers who will develop models for new phenomena, incorporate transport processes into novel applications and develop better analytical/numerical solution procedures for the transport equations.

I learnt the subject of transport processes as a student at IIT Madras and at Cornell University, but I began to understand the subject only after teaching for many years, and after research discussions with students who often had a better understanding than me. For that, I am grateful to the generations of students who have helped me in this journey from learning to understanding.

I would like to thank Prof. Kesava Rao for his detailed review and critical comments on the section on binary diffusion. My colleagues in the Department of Chemical Engineering at the Indian Institute of Science welcomed me into this department many years ago, and guided and supported me as I progressed in my career. They provided a congenial atmosphere and freedom to think and explore, and they did not burden me with even a fair share of administrative responsibilities. To them, and to the administrative staff who have assisted me many times beyond the call of duty, I am grateful. I would like to thank my family, Nandini, Radha and Sudha, for their support and patience while I was writing the book, and it appeared the process would never end.