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978-0-999-28298-4 — Algorithms Illuminated
Tim Roughgarden
Frontmatter
More Information

Algorithms Illuminated

(Omnibus Edition)

In *Algorithms Illuminated*, Tim Roughgarden teaches the basics of algorithms in the most accessible way imaginable, with thorough coverage of asymptotic analysis, graph search and shortest paths, data structures, divide-and-conquer algorithms, greedy algorithms, dynamic programming, and NP-hard problems. Readers of this book will become better programmers; sharpen their analytical skills; learn to think algorithmically; acquire literacy with computer science's greatest hits; and ace their technical interviews.

Tim Roughgarden is a Professor of Computer Science at Columbia University. His research, teaching, and expository writings have been recognized by a Presidential Early Career Award for Scientists and Engineers, the ACM Grace Murray Hopper Award, the EATCS-SIGACT Gödel Prize, a Guggenheim Fellowship, the INFORMS Lanchester Prize, and a Tau Beta Pi Teaching Award. His other books include *Twenty Lectures on Algorithmic Game Theory* and *Beyond the Worst-Case Analysis of Algorithms*.

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More Information

Praise for *Algorithms Illuminated*

“*Algorithms Illuminated* is academic gold for both students and instructors. The most incredible thing about this book is the way it reaches different types of learners. Beginning students love the approach of getting exposed to complex material slowly and methodically, with lots of intuition and illustrations; advanced students can focus their energy on the formal treatment, going back to the intuition as needed. I love this book because it makes very difficult concepts accessible to everyone.”
-Thomas Cook, *U.S. Military Academy at West Point*

“Tim Roughgarden’s *Algorithms Illuminated* is a well-crafted, thorough, and engaging presentation of algorithms on a wide range of topics. The chapters on NP-hard problems are particularly remarkable, as they overturn the conventional wisdom that this topic is too hard for students to understand. I have found that my students really benefit from this approach and strongly recommend this book to others.”
-Avraham Leff, *Yeshiva College*

“In my experience, students find *Algorithms Illuminated* much more relatable and motivating to read than other algorithms books. The presentation style is very modern and honest: complicated ideas are broken down into digestible chunks with the explicit recognition that the reader is indeed facing deep content; pseudocode, proofs, and mathematical techniques are presented truthfully without hiding "technical/boring" details; and every problem is well motivated with the actual reasons why computer scientists care about it. The reader gets the feeling that the book is talking directly to them and inviting them to be part of the story, not just a guest who looks at the highlights and moves on.”
-Saray Shai, *Wesleyan University*

“This fabulous book is an ideal introduction to the design and analysis of algorithms. *Algorithms Illuminated* stands out for its accessible and readable style, with plenty of examples, quizzes and problems for students to check their understanding. The clarity of the exposition brings out the beautiful ideas at the core of the subject. I highly recommend *Algorithms Illuminated* to anyone starting out with algorithms!”
-Mary Wootters, *Stanford University*

“Look through the Table of Contents and you might conclude that this is just another algorithms book. Don’t be fooled. What makes this book special—what makes this book the first of its kind—is Tim Roughgarden’s singular ability to weave algorithm design with pedagogical design. Learners need opportunities to check their understanding at key points, to study examples, to see algorithms in contexts that they care about, to confront the needed mathematical background buffered by these motivating contexts. It’s all here, carried along by Roughgarden’s enthusiasm not only for algorithms but also for the people who want to learn them.”
-Daniel Zingaro, *University of Toronto*

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Frontmatter
More Information

To Emma and Marz

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More Information

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Frontmatter
More Information

Contents

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More Information

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Frontmatter
More Information

Preface

This book has only one goal: *to teach the basics of algorithms in the most accessible way possible*. Think of it as a transcript of what an expert algorithms tutor would say to you over a year of one-on-one lessons. This book is inspired by my online algorithms courses that have been running regularly since 2012, which in turn are based on an undergraduate course that I taught many times at Stanford University. People of all ages, backgrounds, and walks of life are well represented in these online courses, with large numbers of students (high-school, college, etc.), software engineers (both current and aspiring), scientists, and professionals hailing from all corners of the world.

What We'll Cover

Algorithms Illuminated will transform you from an algorithms newbie (say, a rising third-year undergraduate) to a seasoned veteran with expertise comparable to graduates of the world's best computer science master's degree programs. Specifically, this book will help you master the following topics.

Asymptotic analysis and big-O notation. Asymptotic notation provides the basic vocabulary for discussing the design and analysis of algorithms. The key concept here is "big-O" notation, which is a modeling choice about the granularity with which we measure the running time of an algorithm. We'll see that the sweet spot for clear high-level thinking about algorithm design is to ignore constant factors and lower-order terms, and to concentrate on how an algorithm's performance scales with the size of the input.

Divide-and-conquer algorithms and the master method. There's no silver bullet in algorithm design, no single problem-solving method that cracks all computational problems. However, there are a few general algorithm design techniques that find successful application across a range of different domains. In the "divide-and-conquer" technique, the key idea is to break a problem into smaller subproblems, solve the subproblems recursively, and then quickly combine their solutions into one for the original problem. We'll see fast divide-and-conquer algorithms for sorting, integer and matrix multiplication, and a basic problem in computational geometry. We'll also cover the master method, which is a powerful tool for analyzing the running time of divide-and-conquer algorithms.

Randomized algorithms. A randomized algorithm "flips coins" as it runs, and its behavior can depend on the outcomes of these coin flips. Surprisingly often, randomization leads to simple, elegant, and practical algorithms. Among other randomized algorithms, we'll describe and analyze in detail the canonical example of randomized QuickSort.

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Frontmatter
More Information

Sorting and selection. As a byproduct of studying the first three topics, we'll learn several famous algorithms for sorting and selection, including MergeSort, QuickSort, and linear-time selection (both randomized and deterministic). These computational primitives are so blazingly fast that they do not take much more time than that needed just to read the input. This book will help you cultivate a collection of such "for-free primitives," both to apply directly to data and to use as the building blocks for solutions to more difficult problems.

Graph search and applications. Graphs model many different types of networks, including road networks, communication networks, social networks, and networks of dependencies between tasks. Graphs can get complex, but there are several blazingly fast primitives for reasoning about graph structure. We begin with linear-time algorithms for searching a graph, with applications ranging from network analysis to task sequencing.

Shortest paths. In the shortest-path problem, the goal is to compute the best route in a network from point A to point B. The problem has obvious applications, like computing driving directions, and also shows up in disguise in many more general planning problems. We'll generalize one of our graph search algorithms and arrive at Dijkstra's famous shortest-path algorithm.

Data structures. This book will turn you into an educated client of several different data structures for maintaining an evolving set of objects with keys. The primary goal is to develop your intuition about which data structure is the right one for your application. The optional advanced sections provide guidance in how to implement these data structures from scratch.

We first discuss heaps, which can quickly identify the stored object with the smallest key and are useful for sorting, implementing a priority queue, and implementing Dijkstra's algorithm in near-linear time. Search trees maintain a total ordering over the keys of the stored objects and support an even richer array of operations. Hash tables are optimized for super-fast lookups and are ubiquitous in modern programs. We'll also cover the bloom filter, a close cousin of the hash table that uses less space at the expense of occasional errors, and the union-find (disjoint-set) data structure.

Greedy algorithms and applications. Greedy algorithms solve problems by making a sequence of myopic and irrevocable decisions. For many problems, they are easy to devise and often blazingly fast. Most greedy algorithms are not guaranteed to be correct, but we'll cover several killer applications that are exceptions to this rule. Examples include scheduling problems, optimal compression, and minimum spanning trees of graphs.

Dynamic programming and applications. Few benefits of a serious study of algorithms rival the empowerment that comes from mastering dynamic programming. This design paradigm takes a lot of practice to perfect, but it has countless applications to problems that appear unsolvable using any simpler method. Our dynamic programming boot camp will double as a tour of some of the paradigm's killer applications, including the knapsack problem, the Needleman-Wunsch genome sequence alignment algorithm, Knuth's algorithm for optimal binary search trees, and the Bellman-Ford and Floyd-Warshall shortest-path algorithms.

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Frontmatter
More Information

Algorithmic tools for tackling NP-hard problems. Many real-world problems are "NP-hard" and appear unsolvable by always-correct and always-fast algorithms. When an NP-hard problem shows up in your own work, you must compromise on either correctness or speed. We'll see techniques old (like greedy algorithms) and new (like local search) for devising fast heuristic algorithms that are "approximately correct," with applications to scheduling, influence maximization in social networks, and the traveling salesman problem. We'll also cover techniques old (like dynamic programming) and new (like MIP and SAT solvers) for developing correct algorithms that improve dramatically over exhaustive search; applications here include the traveling salesman problem (again), finding signaling pathways in biological networks, and television station repacking in a recent and high-stakes spectrum auction in the United States.

Recognizing NP-hard problems. This book will also train you to quickly recognize an NP-hard problem so that you don't inadvertently waste time trying to design a too-good-to-be-true algorithm for it. You'll acquire familiarity with many famous and basic NP-hard problems, ranging from satisfiability to graph coloring to the Hamiltonian path problem. Through practice, you'll learn the tricks of the trade in proving problems NP-hard via reductions.

For a more detailed look into the book's contents, check out the "Upshot" sections that conclude each chapter and highlight the most important points. The "Field Guide to Algorithm Design" on page 630 provides a bird's-eye view of how to apply the algorithmic toolbox you'll acquire from this book to problems that you encounter in your own work.

The starred sections of the book are the most advanced ones. The time-constrained reader can skip these sections on a first reading without any loss of continuity.

Skills You'll Learn

Mastering algorithms takes time and effort. Why bother?

Become a better programmer. You'll learn several blazingly fast subroutines for processing data as well as several useful data structures for organizing data that you can deploy directly in your own programs. Implementing and using these algorithms will stretch and improve your programming skills. You'll also learn general algorithm design paradigms that are relevant to many different problems across different domains, as well as tools for predicting the performance of such algorithms. These "algorithmic design patterns" can help you come up with new algorithms for problems that arise in your own work.

Sharpen your analytical skills. You'll get lots of practice describing and reasoning about algorithms. Through mathematical analysis, you'll gain a deep understanding of the specific algorithms and data structures that this book covers. You'll acquire facility with several mathematical techniques that are broadly useful for analyzing algorithms.

Think algorithmically. After you learn about algorithms, you'll start seeing them everywhere, whether you're riding an elevator, watching a flock of birds, managing your investment portfolio, or even watching an infant learn. Algorithmic thinking is increasingly useful and prevalent in disciplines outside of computer science, including biology, statistics, and economics.

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Literacy with computer science's greatest hits. Studying algorithms can feel like watching a highlight reel of many of the greatest hits from the last sixty years of computer science. No longer will you feel excluded at that computer science cocktail party when someone cracks a joke about Dijkstra's algorithm. After reading this book, you'll know exactly what they mean.

Ace your technical interviews. Over the years, countless students have regaled me with stories about how mastering the concepts in this book enabled them to ace every technical interview question they were ever asked.

Using this Book in a Course

All of this book's material has been battle-tested in a university setting—by yours truly at Stanford, and by many instructors at other schools. Parts I–III are tailor-made for serving as the primary text in an introductory undergraduate course on algorithms and data structures, focusing on the basics (Part I), graph algorithms and data structures (Part II), and greedy algorithms and dynamic programming (Part III). For example, I cover 75-80% of this content over nineteen 75-minute lectures; a semester-long course could accommodate the rest of it, or selected topics from Chapter 19 about NP-hard problems. Parts III and IV of the book form an ideal basis for a traditional master's level course on basic and advanced algorithms, emphasizing greedy algorithms and their applications, dynamic programming and its applications, the recognition of NP-hard problems, and algorithmic tools for tackling such problems.

Many chapters of this book are logically independent of each other. For example, the design paradigms of divide-and-conquer algorithms (Chapters 3–6), greedy algorithms (Chapters 13–15), and dynamic programming (Chapter 16–18) can be covered in any order. Similarly, data structures (Chapters 10–12) can be taught before or after basic graph algorithms (Chapters 7–9), with the exception of the heap-based implementation of Dijkstra's algorithm in Section 10.4.

As for prerequisites, students in an algorithms course generally have at least a little background in programming (including, for example, the use of arrays and recursion) and in mathematical reasoning (such as proofs by induction and by contradiction). Readers can level up their programming and mathematical skills with any number of free online resources (see `www.algorithmsilluminated.org` for specific recommendations). Appendices A and B also offer quick reviews of induction and discrete probability, respectively. Alternatively, readers without any programming experience can learn the basics of algorithm design and analysis from this book at the level of pseudocode descriptions (if not concrete implementations), and those with little mathematical background can focus squarely on algorithm design techniques (if not detailed algorithm analysis).

I've recorded over 200 YouTube videos that cover in detail every section of the book, as well as additional advanced content (on Karger's random contraction algorithm, path compression in disjoint set data structures, Johnson's all-pairs shortest-path algorithm, and more). These videos, which are catalogued at `www.algorithmsilluminated.org`, can serve an instructor in three different ways: (i) as part of a flipped classroom, with students watching lecture videos in advance of class time, which can then be used for discussion and problem-solving exercises; (ii) as a way to help students fill in missing prerequisites without

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Frontmatter
More Information

taking up class time; and (iii) as supplementary material above and beyond what is covered during class time (perhaps for an honors section or extra credit).

Additional Features and Resources

This book is based on online courses that are currently running on the Coursera and EdX platforms. I've made several resources available to help you replicate as much of the online course experience as you like.

Videos. If you're more in the mood to watch and listen than to read, check out the YouTube video playlists available at `www.algorithmsilluminated.org`. These videos feature yours truly teaching all the topics in this book, as well as additional advanced topics. I hope they exude a contagious enthusiasm for algorithms that, alas, is impossible to replicate fully on the printed page.

Quizzes. How can you know if you're truly absorbing the concepts in this book? Over 100 quizzes with solutions and explanations are scattered throughout the text; when you encounter one, I encourage you to pause and think about the answer before reading on.

End-of-chapter problems. At the end of each chapter, you'll find several relatively straightforward questions that test your understanding, followed by harder and more open-ended challenge problems. Hints or solutions to all of these problems (as indicated by an "*(H)*" or "*(S)*," respectively) are included at the end of the book. Readers can interact with me and each other about the end-of-chapter problems through the book's discussion forum (see below).

Programming problems. Most of the chapters conclude with suggested programming projects whose goal is to help you develop a detailed understanding of an algorithm by creating your own working implementation of it. Data sets, along with test cases and their solutions, can be found at `www.algorithmsilluminated.org`.

Discussion forums. A big reason for the success of online courses is the opportunities they provide for participants to help each other understand the course material and debug programs through discussion forums. Readers of this book have the same opportunity via the forums available at `www.algorithmsilluminated.org`.

Acknowledgments

This book would not exist without the passion and hunger supplied by the hundreds of thousands of participants in my algorithms courses over the years. I am particularly grateful to those who supplied detailed feedback on earlier drafts: Tonya Blust, Yuan Cao, Lauren Cowles, Leslie Damon, Tyler Dae Devlin, Roman Gafiteanu, Carlos Guia, Blanca Huergo, Jim Humelsine, Tim Kearns, Vladimir Kokshenev, Bayram Kuliyev, Patrick Monkelban, Kyle Schiller, Nissanka Wickremasinghe, Clayton Wong, Lexin Ye, Daniel Zingaro, several anonymous reviewers, and many pseudonymous contributors to the book's discussion forums. Thanks also to several experts who provided technical advice: Amir Abboud, Vincent Conitzer, Christian Kroer, Aviad Rubinstein, and Ilya Segal.

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I always appreciate suggestions and corrections from readers. These are best communicated through the discussion forums mentioned above.

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