

Index

Bold face page numbers indicate pages with background information about the entry, e.g., a detailed explanation or reference, while page numbers in normal type indicate a textual reference.

- algebraic function, 259
 algebraic geometry, 83, 84, 231
 algebraic number, **121**
 Arithmetic-Mean–Geometric-Mean (AM–GM) Inequality, **56**, 95, 153, 253, 254, 274
 automata, 108, 122, 123
 Axiom of Choice, **112**
- Bateman-Horn Conjecture, 279
 Beatty sequences, **179**, 180
 Beatty’s Theorem, **179**, 216
 Bernoulli numbers, 184, **225**
 Bessel functions, **63**, **236**
 big- O notation, **89**, 90, 107, 111, 117, 118, 134, 204, 208, 258, 275
 bijection, 21, 53, 82, 123, 124, 140, 141, 173, 174, 195, 223, 229
 binary representation, 11, 17, 39, 43, 107, 108, 156, 157, 188
 Birkhoff-von Neumann Theorem, **68**
 block matrix, 75
 Brauer group, 169
- Carmichael’s lambda function, **57**
 Catalan numbers, **206**
 Cauchy’s Lemma, **143**
 Cauchy’s Theorem, **282**, 283
 Cauchy-Schwarz Inequality, **55**
 Cayley diagram, **127**
 Cayley digraph, **127**
 Cayley-Hamilton Theorem, 63, **122**, 199, 228
 Cayley-Menger determinant, **188**
 centroid, 40, 47, 129, 130, **130**, 131, 132, 233
 Chain Rule, 139
 change of variables, 183, 184
 character, 64, 150, 151
 Chebychev’s Inequality, **55**
 chess, 158
- Chinese Remainder Theorem, 58, 202, 257
 circuit, **126**
 circulant matrix, **98**, 99, 276
 class number, 94
 combinatorial games, 212, 214
 compactness, 44, 49, 104, 134, 195, 230, 266
 companion matrix, 268
 Comparison Test, **97**, 282
 compass and straightedge, 218
 complementary sequences, **179**, 180
 complex numbers, 143
 concavity of functions, 48, 55, 132–134, 197, 251
 content (of a polynomial over \mathbb{Z}), **259**
 continued fraction, **74**, 212
 continuity, 4, 7, 12, 13, 20, 25, 29, 32, 34, 42, 49, 56, 68, 79, 80, 85, 111, 113, 124, 130, 132–134, 138, 139, 143, 155, 156, 166, 176, 184, 220–222, 242, 251, 272, 274, 275, 292, 293
 contour integration, 59, 283
 contraction mapping, 241
 convergence, 10, 37, 41, 42, 61, 62, 72, 80, 89, 97, 110, 111, 125, 162, 183, 191, 194, 195, 204, 221, 222, 228, 234–236, 259, 265, 282–284, 291
 convexity, 13, 14, 26, 55, 68, 118, 120, 128, 130, 131, 134, 159, 160, 230, 231, 270
 cosine law, *see* Law of Cosines
 countability, 12, 44, 111, 112, 120, 121, 195, 219
 cycle, **126**
- de Moivre’s Theorem, 35, **58**
 derangement, **99**
 Descartes’ Rule of Signs, 60, **60**, 198
 design theory, 294
 determinants, xi, 10, 18, 24, 32, 45, 70, 97–99, 164, 188, 193, 200, 211, 276
 diagonalization, 227
 Dickson’s Conjecture, **279**
 difference operator, 259

- differential equations, 36–38, 63, 70, 78, 89, 93, 110, 111, 124, 207, 235, 239, 243, 270, 275
- dimension, 134, 167, 187, 209, 215, 216, 266, 277
- directed multigraph, **126**, 127
- Dirichlet L -function, 95
- Dirichlet density, **280**
- Dirichlet's Theorem, **280**
- discriminant, 74, 178, 198, 260, 261, 273
- divisibility of binomial coefficients, 25, 147, 220, 238, 290, 293
- division algorithm, 163
- Dominated Convergence Theorem, 236, 283
- doubly stochastic matrices, **68**
- dynamical systems, 85, 159, **163**, 212, 222
- eigenvalue, 49, **63**, 93, 98, 99, 167, 184, 199, 200, 207, 227–229, 241, 242, 276–277
- eigenvector, 9, 38, 45, 49, **63**, 93, 99, 184, 207, 227, 241, 242, 270, 276
- Eisenstein integers, **91**
- elliptic curve, **258**
- Euler ϕ -function, 35, **57**, 58, 247, 267, 280
- Euler line, **233**
- Euler's Theorem, **57**, 58
- Euler-Maclaurin summation formula, **225**, 227
- Eulerian circuit, **126**, 127
- Eulerian path, **126**, 127
- Extreme Value Theorem, **184**, 266
- Farey series, **182**
- Fermat's Little Theorem, **86**, 202
- Fibonacci numbers, 40, **93**, 117, 123, 124, 139, 173, 222, 223, 252
- field, xi, 5, 7, 8, 11, 16, 74, 83, 84, 86, 93–95, 107, 108, 128, 148, 158, 164, 167, 178, 261
- field of complex numbers, xi
- field of rational numbers, xi
- field of real numbers, xi
- finite directed multigraph, **126**
- finite field, xi, 8, 11, 16, 86, 107, 108, 148, 151, 158, 203, 261
- first order sentence, **95**, 96
- Fourier analysis, 58, 184, 216
- Fourier series, 58, 216
- Fresnel integrals, 284
- functional equation, 92, 110, 142, 154, 178, 220, 221, 292
- Fundamental Theorem of Calculus, 138, 176
- Gabriel's Horn, **138**
- Gall-Peters projection, 255
- games, 11, 20, 24, 27, 47, 102, 103, 144, 158, 180, 182, 212–214, 233, 234
- Gauss sum, **150**, 151
- Gauss's Lemma, 86, 258, **259**
- Gauss-Lucas Theorem, *see* Lucas' Theorem
- Gaussian elimination, 97
- Gaussian integers, **150**
- Gegenbauer polynomials, **270**
- generalized hypergeometric functions, *see* hypergeometric functions
- Generalized Riemann Hypothesis (GRH), vii, 94, 95
- generating function, 47, 82, **83**, 93, 145, 158, 161, 239, 246, 293
- generating functions, 266
- geometric probability, 159, 160, 182
- geometric transformations, 254
- geometry, 39, 83
- gradient, 46, **68**, 217
- graph theory, 40, 126, 242
- greedy algorithms, 36, 48, 252
- grep, 123
- group theory, 3, 11, 14, 27, 57, 63, 99, 109, 120, 126, 143, 168–170, 178, 195, 202, 237, 238
- harmonic series, 61, 164, 244
- height function, **258**
- Hensel's Lemma, **72**, 261
- Hilbert metric, 241
- Hilbert's Seventeenth Problem, 264
- Hilbert's Tenth Problem, 96
- hypergeometric functions, **283**, 284
- Hypothesis H, 279
- Inclusion-Exclusion Inequality, 201
- Inclusion-Exclusion Principle, 99, 126, 201
- indegree, **126**
- inequalities, 32, 49, 55, 56, 65, 88, 89, 95, 103, 104, 132, 139, 151, 153, 162, 180, 181, 201, 218, 219, 224, 252, 253, 271, 274
- inequalities of integrals, 49, 85, 139, 225, 274
- Integral Comparison Test, **89**
- integral domain, **107**
- integration by parts, 39, 47, 50, **110**, 132, 234, 235, 270, 273, 281, 282

- Intermediate Value Theorem, **102**, 128, 130, 251, 278, 291
- International Mathematical Olympiad, 74, 87, 121, 144, 158, 179, 190, 220, 265, 293
- isometric embedding, **211**
- Jacobi sum, **150**, 151
- Jensen's Inequality, **55**, 56
- Jordan canonical form, 63
- Kummer's Theorem, **238**, 290, 293
- L'Hôpital's Rule, **152**, 156, 166, 269
- Lagrange interpolation, 259, **260**, 289
- Lagrange multipliers, **217**
- Lagrange's Theorem, **57**, **268**
- Lambert equal-area cylindrical projection, 255
- Laplace transform, **236**
- lattice polygon, 118
- lattices, 91, 111, 118–120, 208, 209
- Laurent polynomial, 35, **59**
- Laurent series, 259
- Law of Cosines, 186
- Law of Sines, 269, **286**
- Legendre polynomial, 270
- Legendre symbol, 86, **149**
- Leibniz's formula, 43, **182**, 183
- lexical scanner, 123
- lg, *see* logarithm
- Lie group, **143**
- Limit Comparison Test, **89**
- linear algebra, 166
- linear operator, 43, 184
- linear recursion, 38, 92, **93**, 116, 117, 172, 199, 207, 208, 246, 252, 265, 267
- linear recursive relation, 199
- linear recursive sequence, 38, 92, **93**, 116, 117, 172, 246, 252, 265, 267
- linear transformation, 191, 192, 277
- little-*o* notation, **89**, 90, 92, 96, 103, 279
- ln, *see* logarithm
- Local Central Limit Theorem, 208
- log, *see* logarithm
- logarithm, **xi**
- logarithmic derivative, 239
- Lucas numbers, **212**
- Lucas' Theorem, **137**, 270
- Macdonald's function, **63**
- Markov equation, **74**
- Markov process, 207
- Mean Value Theorem, 117, **198**
- measure theory, 120, 160, 270
- Monotone Convergence Theorem, 183, 234
- Newton polygon, **128**
- Newton's method, 72
- Newton-Puiseux series, **260**
- Nim, **213**
- notation, **xi**
- number theory, 72, 120, 148, 183
- O*, *see* big-*O* notation
- o*, *see* big-*o* notation
- Olympiad, 74, 87, 92, 101, 121, 144, 158, 161, 179, 190, 214, 220, 247, 265, 270, 293
- orthogonal polynomials, **270**
- orthogonality relations for characters, **64**
- outdegree, **126**
- p*-adic absolute value, **128**
- p*-adic gamma function, 147
- p*-adic logarithm function, **xi**
- p*-adic numbers, xi, **128**, 147, 148
- p*-adic valuation, **245**, 257
- p*-group, **169**
- parity, 40, 45, 85, 87, 118, 141, 212, 237, 256, 258
- partition, 24, 140, 141, 179, 209, 214, 216
- path, **126**
- Pell's equation, **279**
- permutation matrix, **68**
- Perron-Frobenius Theorem, 184, 207, 241, **241**, 242
- Pick's Theorem, 40, 118, **119**
- Pigeonhole Principle, 43, 50, **96**, 118, 175, 193, 195, 215, 248, 267, 289, 294
- Pochhammer symbol, **284**
- polynomial ring, **xi**
- Postage Stamp Theorem, **143**
- Power Mean Inequality, **55**, 56
- primitive root, 202, **247**
- probability, 2, 62, 115, 159, 160, 182, 183
- product rule, 9, 88, 136
- Puiseux series, **259**
- Pythagorean Theorem, 85, 287, 294
- Pythagorean triple, **85**
- quadratic form, 95
- quadratic reciprocity, 86
- radius of curvature, 210
- Ramsey theory, **219**
- random walk, 207, **208**, 209, 241
- Ratio Test, **111**, 125, 235, 284

- recursion, 38, 46–48, 85, 92, 93, 116, 117, 165, 172, 199, 207, 208, 223, 229, 230, 239, 246, 252, 265, 267, 270
- reduced totient function, *see* Carmichael's lambda function
- reflection trick, 48, **254**
- règle des nombres moyens, 180
- representation theory, 64, 99, 168, 169
- residue theorem, 59
- Riemann sum, 12, 114, **115**, 134
- Riemann zeta function, **183**
- ring, **xi**
- ring of integers, **xi**
- Rolle's Theorem, 42, 50, **136**, 137, 155, 198, 291
- Rouché's Theorem, **102**, 231
- set theory, 173
- Siegel zero, 95
- sine law, *see* Law of Sines
- sketching, 9, 43, 88
- spherical cap, 255
- Sprague-Grundy theory, **214**
- stationary phase approximation, **285**
- Stirling's approximation, 208, **225**
- Stokes' Theorem, 37, **79**
- strongly connected, **126**, 242
- sum of two squares, 33, 150, 263, 278, 279
- Sylow subgroup, 238
- symmetry, 35, 37, 38, 42, 45, 71, 80, 88, 109, 112, 117, 118, 136, 160, 162, 169, 192, 208, 281
- Taylor series, 38, 45, 48, 89, 103, 104, **107**, 156, 204, 257, 275
- Taylor's Formula, **155**, 257, 273
- Taylor's Theorem, **133**
- Thue-Morse sequence, **158**
- Thue-Siegel Theorem, **261**
- transcendental number, **121**, 158
- Trapezoid Rule, 132, 243
- triangle inequality, 103, 218, 252
- trigonometric substitution, 37, 46, 50, 85, 192, 217, 292
- Turing machine, 95
- Twin Prime Conjecture, **279**
- ultraspherical polynomials, 270
- unique factorization domain, 59, 259
- USA Mathematical Olympiad, 214, 247
- Vandermonde determinant, **70**, 98, 276
- Vandermonde's identity, **82**, 83, 145
- vectors, 46, 93, 159, 186, 187, 218, 230, 231, 233, 276
- Venn diagram, 53
- weakly connected, **126**
- Weierstrass M -test, **111**
- Weierstrass's Theorem, **111**
- Weil Conjectures, vii, **151**, 261
- Well Ordering Principle, **112**
- well-ordered set, **112**, 194, 195
- Weyl's Equidistribution Theorem, **96**, **216**
- winding number, 230, 231
- Wolstenholme's Theorem, **147**, 245
- Wythoff's game, **180**
- Zorn's Lemma, 111, **112**, 195