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0883850370 - From Error-Correcting Codes Through Sphere Packings to Simple Groups

Thomas M. Thompson

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