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# Noncommutative Rings

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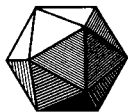
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TO THE MEMORY OF MY FATHER

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## PREFACE

This book is not intended as a treatise on ring theory. Instead, the intent here is to present a certain cross-section of ideas, techniques and results that will give the reader some inkling of what is going on and what has gone on in that part of algebra which concerns itself with noncommutative rings. There are many portions of great importance in the theory which are not touched upon or which are merely mentioned in passing. On the other hand there is a rather detailed treatment given to some aspects of the subject.

While the account given here is not completely self-contained, to follow it does not require a great deal beyond a good first course in algebra. Perhaps I should spell out what I would expect in such a course. To begin with one should have been introduced to some of the basic structures of algebra—groups, rings, fields, vector spaces—and to have seen some of the basic theorems about them. One would want a good familiarity with homomorphisms, the early homomorphism theorems, quotient structures and the like. One should have learned with some thoroughness linear algebra—the fundamental theorems about linear transformations on a vector space. This type of material can be found in many books, for instance, Birkhoff and MacLane *A Survey of Modern Algebra* or my book *Topics in Algebra*.

Beyond these standard topics cited above I shall make frequent use of results from the theory of fields. All these can be found in the chapter on field theory in van der Waerden's *Modern Algebra*. My advice, to the reader not familiar with this material, is to read into a proof until such a result is cited and then to read about the notions arising in van der Waerden's book. Finally, I shall continually use Zorn's Lemma and the axiom of choice.



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A great deal of what is done in this book is based on selected parts of two sets of my notes published in the University of Chicago lecture notes series. Part of this selection and weeding process, polishing and blending together was accomplished in a course I gave at Bowdoin College, under the auspices of the Mathematical Association of America, in the summer of 1965 to a group of mathematicians teaching at various colleges and smaller universities. I should like to thank the participants in that course for their patience and enthusiasm. There are many others I should like to thank, Nathan Jacobson and Irving Kaplansky, for the part they and their work have played in my formation as a mathematician, Shimshon Amitsur for the many pleasant hours spent together working and discussing ring theory and my students, Claudio Procesi and Lance Small, for taking the notes at Bowdoin and for their stimulating comments, suggestions and improvements.

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