

## Contents

<i>Preface</i>	<i>page ix</i>
<b>PART ONE DETERMINISTIC DIFFERENTIAL EQUATIONS</b>	<b>1</b>
<b>1 Linear Analysis</b>	<b>1</b>
1.1 Banach spaces $C^r$ and $L^p$	1
1.2 Hilbert spaces $L^2$ and $H^r$	9
1.3 Linear operators and spectral theory	15
1.4 Fourier analysis	28
1.5 Notes	35
Exercises	36
<b>2 Galerkin Approximation and Finite Elements</b>	<b>40</b>
2.1 Two-point boundary-value problems	40
2.2 Variational formulation of elliptic PDEs	58
2.3 The Galerkin finite element method for elliptic PDEs	66
2.4 Notes	80
Exercises	83
<b>3 Time-dependent Differential Equations</b>	<b>87</b>
3.1 Initial-value problems for ODEs	88
3.2 Semigroups of linear operators	97
3.3 Semilinear evolution equations	102
3.4 Method of lines and finite differences for semilinear PDEs	107
3.5 Galerkin methods for semilinear PDEs	110
3.6 Finite elements for reaction–diffusion equations	120
3.7 Non-smooth error analysis	124
3.8 Notes	132
Exercises	134
<b>PART TWO STOCHASTIC PROCESSES AND RANDOM FIELDS</b>	<b>137</b>
<b>4 Probability Theory</b>	<b>137</b>
4.1 Probability spaces and random variables	137
4.2 Least-squares approximation and conditional expectation	152

vi	<i>Contents</i>	
	4.3	Convergence of random variables 157
	4.4	Random number generation 164
	4.5	Notes 177
		Exercises 178
<b>5</b>	<b>Stochastic Processes</b>	181
	5.1	Introduction and Brownian motion 181
	5.2	Gaussian processes and the covariance function 189
	5.3	Brownian bridge, fractional Brownian motion, and white noise 193
	5.4	The Karhunen–Loève expansion 199
	5.5	Regularity of stochastic processes 206
	5.6	Notes 214
		Exercises 215
<b>6</b>	<b>Stationary Gaussian Processes</b>	217
	6.1	Real-valued stationary processes 217
	6.2	Complex-valued random variables and stochastic processes 225
	6.3	Stochastic integrals 228
	6.4	Sampling by quadrature 234
	6.5	Sampling by circulant embedding 241
	6.6	Notes 254
		Exercises 254
<b>7</b>	<b>Random Fields</b>	257
	7.1	Second-order random fields 258
	7.2	Circulant embedding in two dimensions 266
	7.3	Turning bands method 283
	7.4	Karhunen–Loève expansion of random fields 293
	7.5	Sample path continuity for Gaussian random fields 304
	7.6	Notes 308
		Exercises 310
	<b>PART THREE STOCHASTIC DIFFERENTIAL EQUATIONS</b>	314
<b>8</b>	<b>Stochastic Ordinary Differential Equations</b>	314
	8.1	Examples of SODEs 315
	8.2	Itô integral 318
	8.3	Itô SODEs 324
	8.4	Numerical methods for Itô SODEs 329
	8.5	Strong approximation 337
	8.6	Weak approximation 346
	8.7	Stratonovich integrals and SODEs 360
	8.8	Notes 365
		Exercises 368
<b>9</b>	<b>Elliptic PDEs with Random Data</b>	372
	9.1	Variational formulation on $D$ 374

Cambridge University Press

978-0-521-89990-1 - An Introduction to Computational Stochastic Pdes

Gabriel J. Lord, Catherine E. Powell and Tony Shardlow

Table of Contents

[More information](#)

	<i>Contents</i>	<i>vii</i>
9.2	Monte Carlo FEM	380
9.3	Variational formulation on $D \times \Omega$	386
9.4	Variational formulation on $D \times \Gamma$	393
9.5	Stochastic Galerkin FEM on $D \times \Gamma$	396
9.6	Stochastic collocation FEM on $D \times \Gamma$	421
9.7	Notes	423
	Exercises	428
<b>10</b>	<b>Semilinear Stochastic PDEs</b>	<b>431</b>
10.1	Examples of semilinear SPDEs	432
10.2	$Q$ -Wiener process	435
10.3	Itô stochastic integrals	445
10.4	Semilinear evolution equations in a Hilbert space	447
10.5	Finite difference method	455
10.6	Galerkin and semi-implicit Euler approximation	459
10.7	Spectral Galerkin method	466
10.8	Galerkin finite element method	471
10.9	Notes	477
	Exercises	479
	<b>Appendix A</b>	<b>482</b>
	<i>References</i>	489
	<i>Index</i>	499