SYNTHETIC CDOs

Modelling, Valuation and Risk Management

Credit derivatives have enjoyed explosive growth in the last decade. One of the most important assets in this industry is synthetic Collateralised Debt Obligations (synthetic CDOs). This book describes the state-of-the-art in quantitative and computational modelling of these instruments.

Starting with a brief overview of the structured finance landscape, the book introduces the basic modelling concepts necessary to model and value simple vanilla credit derivatives. Building on this the book then describes in detail the modelling, valuation and risk management of synthetic CDOs. A clear and detailed picture of the behaviour of these complex instruments is built up. The final chapters introduce more advanced topics such as portfolio management of synthetic CDOs and hedging techniques, often not covered in other texts.

Mathematics, Finance and Risk

Editorial Board

Mark Broadie, Graduate School of Business, Columbia University Sam Howison, Mathematical Institute, University of Oxford Neil Johnson, Centre of Computational Finance, University of Oxford George Papanicolaou, Department of Mathematics, Stanford University

SYNTHETIC CDOs

Modelling, Valuation and Risk Management

CRAIG MOUNFIELD





Shaftesbury Road, Cambridge CB2 8EA, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314-321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi - 110025, India

103 Penang Road, #05-06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of Cambridge University Press & Assessment, a department of the University of Cambridge.

We share the University's mission to contribute to society through the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org Information on this title: www.cambridge.org/9780521897884

© C. C. Mounfield 2009

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press & Assessment.

First published 2009

A catalogue record for this publication is available from the British Library

Library of Congress Cataloging-in-Publication data Mounfield, Craig, 1969– Synthetic CDOs : modelling, valuation and risk management / Craig Mounfield. p. cm. – (Mathematics, finance and risk) Includes bibliographical references and index. ISBN 978-0-521-89788-4 (hbk.) 1. Collateralized debt obligations. I. Title. II. Series. HG6024.A3M69 2009 332.63 '2 – dc22 2008043035

ISBN 978-0-521-89788-4 Hardback

Cambridge University Press & Assessment has no responsibility for the persistence or accuracy of URLs for external or third-party internet websites referred to in this publication and does not guarantee that any content on such websites is, or will remain, accurate or appropriate.

Dedicated to my parents, my wife and my daughter.

Cambridge University Press & Assessment 978-0-521-89788-4 — Synthetic CDOs C. C. Mounfield Frontmatter <u>More Information</u>

Contents

	Preface		<i>page</i> xi
	Ack	xvi	
1	A p	1	
	1.1	Introduction	1
	1.2	Securitisation and tranching	2
	1.3	Credit derivative products	6
	1.4	Chapter review	24
2	Moo	lelling of obligor default	25
	2.1	Introduction	25
	2.2	Modelling single-name default as a Poisson process	26
	2.3	Modelling default correlation – fundamental concepts	31
	2.4	Introducing default dependence via copulas	33
	2.5	Rating transition methods for modelling obligor default	36
	2.6	Chapter review	43
3	Valu	nation of credit default swaps	45
	3.1	Introduction	45
	3.2	Overview of vanilla credit default swaps	46
	3.3	Valuation of vanilla CDSs	51
	3.4	Calibration of the survival curve to market observed data	58
	3.5	Risk sensitivities of vanilla CDSs	62
	3.6	Chapter review	65
4	Cree	lit indices	66
	4.1	Introduction	66
	4.2	Description of the credit indices	67
	4.3	Index trading mechanics	69
	4.4	Valuation of credit indices	72
	4.5	Time series analysis of credit indices	73

viii		Contents	
	4.6	Tranched credit index exposures	78
	4.7	Chapter review	80
5	Valı	ation of default baskets	81
	5.1	Introduction	81
	5.2	Brief overview of default baskets	82
	5.3	General valuation principles for default baskets	84
	5.4	Analytic valuation of default baskets in simple	
		limiting cases	86
	5.5	Monte Carlo valuation of default baskets	89
	5.6	Phenomenology of default baskets	93
	5.7	Semi-analytic valuation of default baskets	105
	5.8	Chapter review	108
6	Valı	ation of synthetic CDOs	110
	6.1	Introduction	110
	6.2	Synthetic CDO cashflow mechanics	111
	6.3	Basic principles of synthetic CDO pricing	114
	6.4	Valuation in the standard market model using Monte Carlo	
		simulation	118
	6.5	Valuation in the standard market model using semi-analytic	
		techniques	121
	6.6	Structural models	133
	6.7	Chapter review	135
7	Phe	nomenology of the standard market model	137
	7.1	Introduction	137
	7.2	Baseline case analysed	137
	7.3	Tranche loss statistics	138
	7.4	Analysis of the portfolio loss distribution	142
	7.5	Correlation and maturity sensitivity of the tranche par	
		spread	149
	7.6	Default baskets revisited	158
0	7.7	Chapter review	158
8	Risk	c quantification of synthetic CDOs	160
	8.1	Introduction	160
	8.2	Synthetic CDO risk factors	160
	8.3	Baseline case analysed	162
	8.4	Quantifying credit spread sensitivities – CS01	163
	8.5	Quantifying correlation sensitivities – correlation vega	172
	8.0 07	Quantifying default risk sensitivities – value-on-default (VoD)	1/4
	8.7	Iranche time decay	1//
	8.8	Credit spread value-at-risk (CVaR)	181

Cambridge University Press & Assessment 978-0-521-89788-4 — Synthetic CDOs C. C. Mounfield Frontmatter <u>More Information</u>

		Contents	ix
	8.9	Default value-at-risk (DVaR)	184
	8.10	Chapter review	189
9	Impli	190	
	9.1	Introduction	190
	9.2	Market quoting conventions	191
	9.3	The correlation smile and implied correlation	192
	9.4	The market solution – base correlations	197
	9.5	Chapter review	203
10	Exter	nsions of the standard market model	204
	10.1	Introduction	204
	10.2	Extending the standard market model	205
	10.3	Dynamic portfolio loss models	221
	10.4	Chapter review	222
11	Exoti	c CDOs	224
	11.1	Introduction	224
	11.2	Synthetic CDO ² and CDO ⁿ	225
	11.3	Cashflow CDOs	229
	11.4	Asset backed CDS (ABCDS)	241
	11.5	ABX indices and tranched ABX (TABX) exposures	243
	11.6	Chapter review	247
12	Corre	elation trading of synthetic CDO tranches	249
	12.1	Introduction	249
	12.2	An overview of correlation trading	250
	12.3	Delta hedging of synthetic CDO tranches	258
	12.4	Analysis of common correlation trading strategies	264
	12.5	Credit market dislocations	270
	12.6	Chapter review	276
13	Risk	management of a portfolio of synthetic CDOs	277
	13.1	Introduction	277
	13.2	Set-up of the problem	278
	13.3	Portfolio risk measures	285
	13.4	Description of the sample portfolio	289
	13.5	Basic analysis of the sample portfolio	292
	13.6	Adding new trades to the portfolio	302
	13.7	Origination of synthetic CDOs	305
	13.8	Chapter review	308
14	Hedg	ing simulation of structured credit products	309
	14.1	Introduction	309
	14.2	What is hedging simulation?	310
	14.3	Hedging of structured credit products	313

Cambridge University Press & Assessment 978-0-521-89788-4 — Synthetic CDOs C. C. Mounfield Frontmatter <u>More Information</u>

Х	Contents	
	14.4 Hedging simulation of default baskets	316
	14.5 Hedging simulation of synthetic CDO tranches	320
	14.6 Portfolio exposure measurement	337
	14.7 Chapter review	349
	Appendix A: Explanation of common notation	351
	Appendix B: Simulated annealing	352
	References	357
	Index	364

Preface

This is a book about the modelling, valuation and risk management of synthetic collateralised debt obligations (or synthetic CDOs or simply CDOs for short). Synthetic CDOs are an example of a structured credit product. This is a financial product that takes targeted risk for the purpose of achieving targeted returns. Structured credit products utilise two financial engineering technologies: credit derivatives and asset securitisation. Synthetic CDOs have played an increasingly important role in the expansion of the global credit derivatives market which has grown rapidly since the turn of the century. Indeed, it is estimated that by the end of 2006 the total credit derivative notional amount outstanding was over \$20 trillion (from virtually zero only a decade earlier). Increased trading volumes naturally led to market participants becoming more sophisticated (in terms of their risk/return characteristics and the strategies they employ) as well as to a commensurate increase in the complexity and subtlety of the products available. This in turn drives the evolution of the mathematical and computational models used to value these products. The objective of this book is to collate, summarise and critically assess the current state-of-the-art in quantitative and computational modelling of synthetic CDOs. The key word here is *modelling*; the book is about mathematical models and their properties. This book is not intended to provide detailed descriptions of the business and economic rationales for trading credit derivatives; there are better resources available that describe this and due reference will be given to these sources. It is meant to provide a detailed quantitative description of the modelling techniques currently employed in the marketplace for characterising synthetic CDOs.

It will be assumed that the technical level and experience of the reader is relatively high. Basic financial concepts will not be described in detail (except insofar as when such detail is necessary). Instead reference will be made to the appropriate resources. The use of financial and technical jargon will hopefully be kept to a minimum, although in a specialised, technical text such as this some jargon is inevitable. The rationale for this approach is to ensure the volume is concise and to the point. It is

Cambridge University Press & Assessment 978-0-521-89788-4 — Synthetic CDOs C. C. Mounfield Frontmatter <u>More Information</u>

xii

Preface

intended to describe just enough of the mathematical and computational modelling to enable the reader to understand the relevant issues (along with a discussion of the practical implementation considerations) and help the reader to form their own opinion as to the merits, or otherwise, of the models presented. I will consider the book to be a success if it enables readers to understand the behaviour of models and to build better versions of them. This lean approach will hopefully make the volume attractive to practitioners (who do not always have the time to study a subject in detail) who wish to understand more about the properties of the credit derivative models commonly used in the marketplace. In particular it is envisaged that the volume will be of interest to a range of different types of practitioner.

- Quantitative analysts (quants) and quant developers wanting to understand more about credit modelling and credit derivatives. The book is written with a strong emphasis on models, implementation and understanding of the model behaviour. It is therefore well suited to quants in model validation teams, for example.
- Quantitative risk managers wanting to understand better the models used for valuation, to interpret synthetic CDO risk sensitivities (e.g. spread and correlation sensitivities) and risk manage complex credit portfolios.
- Traders and product controllers seeking a better understanding of the mechanics going on in the black boxes when 'F9' is pressed (and to understand the relative strengths and weaknesses of different models).
- Structurers wanting to understand better the properties of the instruments they are using to construct strategies with specific risk/return characteristics.
- Researchers in academia looking to understand some of the practical issues surrounding the common models used in the marketplace.

The downside to this lean approach is that for less experienced readers the material may at times not give as much explanation as would be liked, or some (basic) concepts are not described fully. However, for the motivated and intelligent reader this should present not a problem but a challenge and (as the author knows from experience) the rewards in terms of deeper understanding are worth the effort.

At the beginning of a project such as writing a book one has a vision as to what the finished product will look like. The vision for this book was that it would be very much model focused, with a strong emphasis on the practical, pragmatic implementation details that are of crucial importance in a live banking environment. This means there is less focus on the 'business' topics of the economics, mechanics and structures of credit derivatives than can be found in other texts. To include this information would have detracted from the core message of models and their properties. Also, when writing a book it is necessary to make compromises and be pragmatic in terms of content. At the beginning of the project one's vision of what will be achieved is vast and expansive. By the end of the project one is simply happy to stumble across the finish line. There are occasions throughout the book

Cambridge University Press & Assessment 978-0-521-89788-4 — Synthetic CDOs C. C. Mounfield Frontmatter <u>More Information</u>

Preface

when more detailed analysis of a particular model or scenario would have been very useful indeed to illustrate a particular point further, but due to time constraints was not included. On these occasions it is suggested that the reader build the models and do the analysis themselves as an exercise.

This leads into the next important point about the approach taken in the text. In the modern world of quantitative finance it is almost impossible to develop models of complex derivative trades that are wholly tractable analytically. It is therefore difficult to separate a model's mathematical description from its actual implementation. When it comes to building models suitable for use within a live investment banking environment the devil really is in the details. Full understanding of a model only comes from implementing it, analysing its properties and understanding its weaknesses. An important objective of this volume, therefore, is to provide not only the mathematical descriptions of the models, but also details of the practical implementation issues. To achieve this objective, liberal use is made of pseudo code to illustrate the implementation of an algorithm. The purpose of this code is to allow the reader to convert quickly a description of a model into the programming environment of their choice (although the author is most familiar with C++, and there may appear to be a bias towards the syntax of this language on occasion).

The volume is structured into three distinct sections. Broadly speaking Chapters 1–3 motivate the main topic, synthetic CDOs, and introduce some of the basic modelling tools necessary to describe them. Chapters 4–10 analyse the mathematical and computational modelling techniques applied to synthetic CDOs. Chapters 11–14 look at more advanced topics in the analysis of synthetic CDOs. Each of the chapters can in principle be read in isolation and each is relatively self-contained. However, there is a clear path from chapter to chapter (which reflects the author's own train of thought), particularly in Chapters 4–10. Reading each chapter sequentially will build a clearer and more coherent picture of the subject matter as a whole, but it is by no means a prerequisite.

In the first part of the book we motivate the study of synthetic CDOs by understanding their importance and usage within the broader credit derivatives marketplace. Chapter 1 provides a brief overview of the credit derivatives market in terms of instruments and introduces the crucial concepts of securitisation and tranching which are the basis of CDO technology. In this first section we also provide some of the basic mathematical building blocks necessary for later chapters. Chapter 2 describes the current market standard modelling methodologies for capturing the arrival of default risk of an obligor. This chapter also introduces the concepts and methods used for the modelling of default correlation, which as we will see is one of the most fundamental concepts in the characterisation of synthetic CDOs (and indeed any multi-name credit derivative). The first section of the book ends with a discussion, in Chapter 3, of the valuation models for the simplest and most

xiii

Cambridge University Press & Assessment 978-0-521-89788-4 — Synthetic CDOs C. C. Mounfield Frontmatter More Information

xiv

Preface

vanilla of credit derivatives – credit default swaps or CDSs. The market for singlename default protection CDSs is extremely liquid and a good understanding of the valuation methods for these basic building blocks is a necessary prerequisite for understanding the more complex multi-name products.¹ For a reader already conversant with single-name credit derivatives, the material in Chapters 1–3 will be familiar. Indeed these chapters are only included in order to provide a reference guide to the concepts underpinning the rest of the book.

The second part of the volume, Chapters 4–10, which is its mathematical and computational core, focuses specifically on the valuation and risk analysis of multiname credit derivatives and synthetic CDOs in particular. Chapter 4 introduces the credit indices that have emerged and evolved over the course of the last few years. The introduction and subsequent trading of these indices has provided enormous impetus to the growth of the credit derivatives market. Chapter 5 then introduces default baskets. In terms of materiality, default baskets are a very small fraction of the overall structured credit marketplace. However, they are the simplest form of multi-name credit derivative and an understanding of their valuation and risk sensitivities can provide substantial insight into the behaviour of more complex synthetic CDOs.

Chapters 6 through 8 develop and analyse the core mathematical models for valuing synthetic CDOs. Chapter 6 describes a number of different methodologies for valuation and, in particular, introduces the current market standard valuation model, the so-called normal copula model. Chapter 7 investigates the fundamental behaviour of the model as certain key parameters are varied systematically. As will be seen in this chapter, the phenomenology of the model is relatively complex and subtle. Chapter 8 analyses the risk sensitivities of the standard market model to variations of input parameters. More importantly this chapter discusses the different risk sensitivity measures such as credit spread 01 (CS01) and value-on-default (VoD) that are necessary to capture and characterise the risk inherent in synthetic CDOs.

The next chapters look at the implications for the standard market model that standardised tranches and the development of a liquid market have had. Initially the market for synthetic CDOs was relatively illiquid and deals were done on a bespoke basis. The introduction of standardised credit indices and the subsequent development of a market for trading tranched exposures to slices of the index provided enormous impetus to the liquidity and volume of trades in single-tranche synthetic CDOs (STCDOs). Eventually the market became sufficiently liquid to allow transparent price discovery for the prices of these standardised index tranches. At this

¹ The main focus of the book is synthetic CDOs. Therefore we will not spend a great deal of time talking about CDSs and other credit derivatives – there are better texts available that describe these products in great detail.

Cambridge University Press & Assessment 978-0-521-89788-4 — Synthetic CDOs C. C. Mounfield Frontmatter <u>More Information</u>

Preface

point the role of the standard model changed; it became a mechanism whereby market participants could express and trade their views on default correlation. Chapter 9 introduces the concepts of implied and base correlations that have been developed to capture implied pricing information from market observed prices. As the prices of instruments become transparent in the open market it is crucially important for the standard model to be able to reproduce these prices accurately. Chapter 10 describes some of the different methodologies that have been developed to allow calibration of models of synthetic CDOs to market observed prices (the so-called 'correlation skew').

The final part of the volume, Chapters 11–14, looks at more advanced topics in the characterisation and analysis of synthetic CDOs. Chapter 11 introduces a number of exotic CDOs. Examples include CDOs with asset backed securities as the underlying pool of obligors as well as CDOs with CDOs as the assets in the underlying pool (so called CDO squareds). Correlation trading is the term used to refer to trading strategies designed to exploit the risk/return characteristics of portfolios of CDO tranches. Chapter 12 analyses the risk/return characteristics of a number of popular CDO trading strategies. Chapter 13 considers extending the models developed thus far for a single-tranche position to a portfolio of tranches and assesses how the risk in the tranche portfolio can be quantified and controlled.

Finally, a natural extension of analysing the static (in time) performance of CDO trading and hedging strategies is to look at the through life performance of the trading strategy. In the pricing of simpler derivatives, the value of the derivative is equal to the cost of the dynamic hedging strategy. If a hedging strategy is good at capturing all the risks a position is exposed to then the overall P/L generated from the process of selling the derivative instrument and rebalancing the hedging portfolio as the market risk factors evolve should be small. If the hedging strategy is not adequate there will be significant P/L leakage. Chapter 14 sets up and analyses a simple hedging simulation of synthetic CDO tranches. This chapter is more speculative in nature than previous chapters as it represents the cutting edge of technology applied to the analysis of complex derivative securities.

xv

Acknowledgements

A book is never written in isolation, and it is a pleasure to acknowledge the contribution that a number of individuals have made to the current text. I would like to thank all the people I have worked with in the Model Validation and Risk Management teams of Credit Suisse and Barclays Capital as well as my co-workers at Cheyne Capital Management. A lot of the experience that is encapsulated in this text is a direct result of day-to-day interactions with my colleagues at these institutions. In particular, I would like to thank Dr Niclas Sandstrom of Barclays Capital and Dr Andrea Petrelli of Credit Suisse for their detailed reading of the original manuscript, and for making numerous suggestions as to how it could be improved.

I would also like to thank my editor at CUP, David Tranah (and all the other staff who have contributed to the bringing to fruition of this project), for providing me with an opportunity to write this book. Finally I would like to acknowledge the contribution of my Ph.D. supervisor Professor Sir S. F. Edwards of the Cavendish Laboratory, Cambridge. The scientific training I received under his tutelage has proven to be of enduring value throughout my career. I hope this text reflects some of what I learnt from him.