

QUANTUM ERROR CORRECTION

Quantum computation and information are among the most exciting developments in science and technology of the last 20 years. To achieve large-scale quantum computers and communication networks it is essential to overcome noise not only in stored quantum information, but also in general faulty quantum operations. Scalable quantum computers require a far-reaching theory of fault-tolerant quantum computation.

This comprehensive text, written by leading experts in the field, focuses on quantum error correction, and thoroughly covers the theory as well as experimental and practical issues. The book is not limited to a single approach, but also reviews many different methods to control quantum errors, including topological codes, dynamical decoupling, and decoherence-free subspaces.

Basic subjects as well as advanced theory and a survey of topics from cutting-edge research make this book invaluable both as a pedagogical introduction at the graduate level and as a reference for experts in quantum information science.

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Contents

<i>List of contributors</i>	<i>page</i> xi
<i>Prologue</i>	xv
<i>Preface and guide to the reader</i>	xix
<i>Acknowledgements</i>	xxi
Part I Background	1
1 Introduction to decoherence and noise in open quantum systems	3
<i>Daniel A. Lidar and Todd A. Brun</i>	
1.1 Introduction	3
1.2 Brief introduction to quantum mechanics and quantum computing	4
1.3 Master equations	26
1.4 Stochastic error models	32
1.5 Conclusions	45
2 Introduction to quantum error correction	46
<i>Dave Bacon</i>	
2.1 Error correction	46
2.2 From reversible classical error correction to simple quantum error correction	48
2.3 The quantum error-correcting criterion	56
2.4 The distance of a quantum error-correcting code	59
2.5 Content of the quantum error-correcting criterion and the quantum Hamming bound	59
2.6 Digitizing quantum noise	60
2.7 Classical linear codes	61
2.8 Calderbank, Shor, and Steane codes	64
2.9 Stabilizer quantum error-correcting codes	65
2.10 Conclusions	76
2.11 History and further reading	76

3	Introduction to decoherence-free subspaces and noiseless subsystems	78
	<i>Daniel A. Lidar</i>	
3.1	Introduction	78
3.2	A “classical decoherence-free subspace”	78
3.3	Collective dephasing decoherence-free subspace	79
3.4	Decoherence-free subspace defined and characterized	81
3.5	Initialization-free decoherence-free subspace	90
3.6	Noiseless subsystems	92
3.7	Initialization-free noiseless subsystems	98
3.8	Protection against additional decoherence sources	101
3.9	Conclusions	102
3.10	History and further reading	102
4	Introduction to quantum dynamical decoupling	105
	<i>Lorenza Viola</i>	
4.1	Motivation and overview	105
4.2	Warm up: bang-bang decoupling of qubit dephasing	107
4.3	Control-theoretic framework	110
4.4	Bang-bang periodic decoupling	113
4.5	The need for advanced decoupling design	119
4.6	Bounded-strength Eulerian decoupling	120
5	Introduction to quantum fault tolerance	126
	<i>Panos Aliferis</i>	
5.1	Quantum circuits and error discretization	127
5.2	Noisy quantum computers	130
5.3	Encoded quantum computation	142
5.4	Coarse-grained noise and level reduction	152
5.5	The quantum accuracy threshold	155
5.6	Assessment	157
5.7	History and further reading	158
	Part II Generalized approaches to quantum error correction	161
6	Operator quantum error correction	163
	<i>David Kribs and David Poulin</i>	
6.1	Introduction	163
6.2	Equivalent conditions for OQEC	165
6.3	Stabilizer formalism for OQEC	169
6.4	Examples	172
6.5	Measuring gauge operators	175
6.6	Bounds for subsystem codes	177
6.7	Unitarily correctable codes	179
7	Entanglement-assisted quantum error-correcting codes	181
	<i>Todd A. Brun and Min-Hsiu Hsieh</i>	
7.1	Introduction	181
7.2	Constructing EAQECCs	184

	<i>Contents</i>	vii
	7.3 Constructing EAQECCs from classical linear codes	195
	7.4 Catalytic QECCs	197
	7.5 Conclusions	199
8	Continuous-time quantum error correction	201
	<i>Ognyan Oreshkov</i>	
	8.1 Introduction	201
	8.2 CTQEC in an encoded basis	204
	8.3 Quantum-jump CTQEC with weak measurements	207
	8.4 Schemes with indirect feedback	213
	8.5 Quantum jumps for Markovian and non-Markovian noise	218
	8.6 Outlook	226
	Part III Advanced quantum codes	229
9	Quantum convolutional codes	231
	<i>Mark Wilde</i>	
	9.1 Introduction	231
	9.2 Definition and operation of quantum convolutional codes	235
	9.3 Mathematical formalism of quantum convolutional codes	238
	9.4 Quantum shift-register circuits	244
	9.5 Examples of quantum convolutional codes	249
	9.6 Entanglement-assisted quantum convolutional codes	253
	9.7 Closing remarks	260
10	Nonadditive quantum codes	261
	<i>Markus Grassl and Martin Rötteler</i>	
	10.1 Introduction	261
	10.2 Stabilizer codes	262
	10.3 Characterization of nonadditive quantum codes	263
	10.4 Construction of nonadditive QECCs	268
	10.5 Quantum circuits	274
	10.6 Conclusions	277
11	Iterative quantum coding systems	279
	<i>David Poulin</i>	
	11.1 Introduction	279
	11.2 Decoding	284
	11.3 Turbo-codes	292
	11.4 Sparse codes	297
	11.5 Conclusion	305
12	Algebraic quantum coding theory	307
	<i>Andreas Klappenecker</i>	
	12.1 Quantum stabilizer codes	307
	12.2 Cyclic codes	317
	12.3 Quantum BCH codes	318
	12.4 Quantum MDS codes	325

13	Optimization-based quantum error correction	327
	<i>Andrew Fletcher</i>	
13.1	Limitation of the independent arbitrary errors model	327
13.2	Defining a QEC optimization problem	328
13.3	Maximizing average entanglement fidelity	331
13.4	Minimizing channel nonideality: the indirect method	336
13.5	Robustness to channel perturbations	338
13.6	Structured near-optimal optimization	340
13.7	Optimization for (approximate) decoherence-free subspaces	346
13.8	Conclusion	347
	Part IV Advanced dynamical decoupling	349
14	High-order dynamical decoupling	351
	<i>Zhen-Yu Wang and Ren-Bao Liu</i>	
14.1	Introduction	351
14.2	Operator set preservation	351
14.3	Dynamical decoupling for multi-qubit systems	353
14.4	Concatenated dynamical decoupling	355
14.5	Uhrig dynamical decoupling	357
14.6	Concatenated Uhrig dynamical decoupling	365
14.7	Quadratic dynamical decoupling	366
14.8	Nested Uhrig dynamical decoupling	367
14.9	Pulses of finite amplitude	368
14.10	Time-dependent Hamiltonians	369
14.11	Randomized dynamical decoupling	372
14.12	Experimental progress	373
14.13	Discussion	374
15	Combinatorial approaches to dynamical decoupling	376
	<i>Martin Rötteler and Pawel Wocjan</i>	
15.1	Introduction	376
15.2	Combinatorial bang-bang decoupling	378
15.3	Combinatorial bounded strength decoupling	391
15.4	Conclusions and future directions	393
	Part V Alternative quantum computation approaches	395
16	Holonomic quantum computation	397
	<i>Paolo Zanardi</i>	
16.1	Introduction	397
16.2	Holonomic quantum computation	398
16.3	HQC with quantum dots	400
16.4	Robustness	403
16.5	Hybridizing HQC and error-avoiding/correcting techniques	406
16.6	Conclusions	407
	Appendix: quantum holonomies	408

	<i>Contents</i>	ix
17	Fault tolerance for holonomic quantum computation	412
	<i>Ognyan Oreshkov, Todd A. Brun, and Daniel A. Lidar</i>	
17.1	Holonomic quantum computation on subsystems	413
17.2	FTHQC on stabilizer codes without additional qubits	415
17.3	Conclusion and outlook	430
18	Fault-tolerant measurement-based quantum computing	432
	<i>Debbie Leung</i>	
18.1	Introduction	432
18.2	Common models for measurement-based quantum computation	433
18.3	Apparent issues concerning fault tolerance in measurement-based quantum computation	442
18.4	Simulation of an operation	442
18.5	Fault tolerance in graph state model	444
18.6	History and other approaches	451
	Part VI Topological methods	453
19	Topological codes	455
	<i>Héctor Bombín</i>	
19.1	Introduction	455
19.2	Local codes	455
19.3	Surface homology	457
19.4	Surface codes	461
19.5	Color codes	471
19.6	Conclusions	480
19.7	History and further reading	480
20	Fault-tolerant topological cluster state quantum computing	482
	<i>Austin Fowler and Kovid Goyal</i>	
20.1	Introduction	482
20.2	Topological cluster states	482
20.3	Logical initialization and measurement	486
20.4	State injection	489
20.5	Logical gates	491
20.6	Topological cluster state error correction	492
20.7	Threshold	496
20.8	Overhead	499
	Part VII Applications and implementations	507
21	Experimental quantum error correction	509
	<i>Dave Bacon</i>	
21.1	Experiments in liquid-state NMR	509
21.2	Ion trap quantum error correction	514
21.3	Experiments using linear optics quantum computation	517
21.4	The future of experimental quantum error correction	518

x	<i>Contents</i>	
22	Experimental dynamical decoupling	519
	<i>Lorenza Viola</i>	
	22.1 Introduction and overview	519
	22.2 Single-axis decoupling	520
	22.3 Two-axis decoupling	532
	22.4 Recent experimental progress and outlook	534
23	Architectures	537
	<i>Jacob Taylor</i>	
	23.1 The principles of fault tolerance	537
	23.2 Memory	539
	23.3 Building gates	543
	23.4 Entangling operations and transport	545
	23.5 Quantum networking	550
24	Error correction in quantum communication	553
	<i>Mark Wilde</i>	
	24.1 Introduction	553
	24.2 Entanglement distillation	554
	24.3 Quantum key expansion	562
	24.4 Continuous-variable quantum error correction	570
	24.5 Implementations of quantum error correction for communication	578
	24.6 Closing remarks	579
	24.7 Historical notes	579
	Part VIII Critical evaluation of fault tolerance	583
25	Hamiltonian methods in quantum error correction and fault tolerance	585
	<i>Eduardo Novais, Eduardo R. Mucciolo, and Harold U. Baranger</i>	
	25.1 Introduction	585
	25.2 Microscopic Hamiltonian models	588
	25.3 Time evolution with quantum error correction	590
	25.4 The threshold theorem in a critical environment	598
	25.5 The threshold theorem and quantum phase transitions	599
	25.6 An example: the simplified spin-boson model	601
	25.7 Conclusions	609
	Some useful results	609
26	Critique of fault-tolerant quantum information processing	612
	<i>Robert Alicki</i>	
	26.1 Introduction	612
	26.2 Fault-tolerant quantum computation	613
	26.3 Fault tolerance and quantum memory	619
	26.4 Concluding remarks	624
	<i>References</i>	625
	<i>Index</i>	657

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xiii

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Prologue

For most of human history we maneuvered our way through the world based on an intuitive understanding of physics, an understanding wired into our brains by millions of years of evolution and constantly bolstered by our everyday experience. This intuition has served us very well, and functions perfectly at the typical scales of human life – so perfectly, in fact, that we rarely even think about it. It took many centuries before anyone even tried to formulate this understanding; centuries more before the slightest evidence suggested that these assumptions might not always hold. When the twin revolutions of relativity and quantum mechanics overturned twentieth-century physics, they also overturned this intuitive notion of the world.

In spite of this, the direct effect at the human scale has been small. Our cars do not run on Szilard engines. Very few freeways, even in Los Angeles, have signs saying “Speed Limit 300,000 km/s.” And human intuition remains rooted in its evolutionary origins. It takes years of training for scientists to learn the habits of thought appropriate to quantum mechanics; and even then, surprises still come along in the areas we think we understand the best.

Technology has transformed how we live our lives. Computers and communications depend on the amazingly rapid developments of electronics, which in turn derive from our understanding of quantum mechanics: we use the microscopic movements of electrons in solids to do work and play games; pulses of coherent light in optical fibers tie the world together. But the theories underlying this technology – computer science and information theory – were built on a fundamentally classical view of the world. These two theories have been fantastically successful – so successful that for many years no one worried (and few even recognized) that they depended on physical assumptions that, at the smallest scales, did not hold. It was only in the past two decades that we realized these theories are only part of a vastly richer subject. This is the subject of quantum information science.

The precursors of quantum information science arrived in the 1980s, building on many years of earlier work on the foundations of quantum mechanics, and spurred by the development of new and powerful experimental methods, such as laser cooling, electromagnetic traps, and optical microcavities. But it was in the 1990s that the revolution began in earnest. Stimulated in large part by the stunning discovery of Peter Shor’s quantum factoring algorithm, dozens and then hundreds of researchers began to examine the properties of quantum theory with a new eye. How could superposition, interference, and entanglement be combined to produce new and better forms of

information processing – new algorithms, new communication protocols, and at the same time new ways of understanding the physical world? The result, nearly two decades later, has been an astonishing avalanche of new ideas. The century-old dog of quantum mechanics has learned a whole host of new tricks, and the output shows no signs of stopping.

In addition to its technological promise, quantum information science has dramatically changed our understanding of quantum physics. It had already been long recognized that there were deep connections between information theory and the laws of thermodynamics. Quantum measurement also seemed to connect information and physics in a fundamental way, where acquiring information about a quantum system was accompanied by unavoidable disturbance. These connections have now spread through the broad field of quantum mechanics. Long-studied systems in optical and condensed matter physics have yielded new insights to an analysis based on the presence and flow of quantum information. Just as quantum effects have been repurposed as resources for information processing, information concepts have provided a new window on physics.

The rapid early development of quantum information science, however, was accompanied by immediate skepticism. The same developments in the foundations of quantum theory that had led to the ideas of quantum information had brought a new appreciation of the power of decoherence – how unavoidable interactions between a quantum system and its external environment cause large-scale superpositions to decay and destroy interference effects. Quantum algorithms were based on idealized models, in which large quantum systems undergo perfect unitary evolution. The larger a quantum system, the more strongly it interacts with its environment, and the more rapidly decoherence will destroy the superpositions and interference effects on which quantum computing depends. Schrödinger's cat, far from being in a superposition of alive and dead, would decohere in an infinitesimal fraction of a second into a probabilistic mixture. The power of decoherence to screen quantum effects at macroscopic levels explains why quantum mechanics was never even suspected for the vast majority of human history.

From a computational point of view, we can think of decoherence as a source of computational errors. In the early days of classical computation, it was feared that large-scale computation might be impossible because of errors: however unlikely they might be, in a sufficiently large computation errors would eventually happen and corrupt the output. To address this concern, John von Neumann studied models of computation and concluded that in fact reliable computations of unlimited size were possible, provided that the rates of error were sufficiently low. The key was error correction: the information in a computer would be stored redundantly using error-correcting codes, and throughout the running of a program the computer would constantly check for errors and correct them when they occur.

Skeptics of quantum computation argued that a similar solution was impossible for a quantum computer, because of two constraints that do not apply to a classical digital computer. First, quantum information cannot be reliably copied – this is the famous no-cloning theorem. And second, errors are detected by measuring the system, but measurements destroy quantum superpositions and eliminate the interference effects on which quantum computers depend. Moreover, unlike digital computers, the allowed states of a quantum system form a continuum. Small imprecisions in the control of the computer could accumulate, eventually derailing the computation. This effect is what has prevented large-scale analog computation.

Fortunately, all these intuitive arguments proved to be incorrect. It is true that quantum information cannot be copied; but it can be spread redundantly over multiple subsystems in a way that

protects it from local errors. It is possible to perform measurements that reveal the presence of errors without probing the actual state of the stored information. And while quantum states form a continuum, this rests on a discrete structure, just like a digital computer; the ability to correct a well-chosen set of discrete errors grants the ability to correct all of their linear combinations as well. Within a few years, the first quantum error-correcting codes were discovered; a theory of the broad class of stabilizer codes, analogous to classical linear codes, had been developed; and threshold theorems had been proven, showing that a quantum computer could be constructed fault-tolerantly in such a way that for sufficiently low rates of error (due to decoherence, imperfect control, or other sources) quantum computations of unlimited length were possible.

Since these early results the field of quantum error correction has dramatically expanded. Brand new codes, and new fault-tolerant techniques, have moved the stringent requirements on error rates closer and closer to reality, even as experiments have reduced the error rates to lower and lower levels. New techniques have also been found to prevent errors – by effectively eliminating the interaction with the environment, in dynamical decoupling; by rendering the system immune to noise with particular symmetries, in decoherence-free subspaces and noiseless subsystems; or by encoding quantum information into naturally robust degrees of freedom, in topological quantum computing.

The world of quantum error correction is already vastly rich, and growing richer all the time. This book will help you enter it, or act as a map and reference for those already knowledgeable. We invite you to come in.

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Preface and guide to the reader

We were inspired to put this book together during the process of organizing the First International Conference on Quantum Error Correction at the University of Southern California (in December 2007, with a sequel in December 2011). With many of the world's foremost experts in the various branches of quantum error correction gathered together in Los Angeles, we solicited chapters on what we thought were the most important topics in the broad field. As editors, we then faced the difficult challenge of integrating material from many expert authors into one comprehensive and yet coherent volume. To achieve this feeling of coherence, we asked all our contributors to work with a single, common notation, and to work with the authors of other chapters in order to minimize overlap and maximize synchronicity. This proved hard to enforce, and while we made every effort to achieve consistency among the different chapters, this goal was surely only partly met. To the extent that the reader discovers inconsistencies, we as editors take full responsibility. The resulting book is not a textbook; for one, it doesn't include any exercises, and figures are not abundant. Moreover, it can only be a snapshot of such a rapidly evolving subject as quantum error correction. Nevertheless, we believe that the basic results in the field are now well enough established that this book, with its extensive index and list of references to the literature, will serve both as a reference for experts and as a guidebook for new researchers, for some years to come.

This book is organized into eight parts, containing a total of 26 chapters by 27 authors, some of whom wrote or co-wrote more than one chapter. Part I (with contributions by Panos Aliferis, Dave Bacon, Todd Brun, Daniel Lidar, and Lorenza Viola) is an introduction; it contains five chapters covering the basics of decoherence and quantum noise, quantum error-correcting codes, dynamical decoupling, decoherence-free subspaces and subsystems, and fault tolerance. Readers interested in a comprehensive introduction to the field, at the graduate course level, will do well to read this first part. It is roughly equivalent to two one-semester courses. Part II (with contributions by Todd Brun, Min-Hsiu Hsieh, David Kribs, Ognian Oreshkov, and David Poulin) presents several important generalizations of quantum error correction (QEC), accounting for additional richness in the Hilbert space structure, and for the possibility of continuously correcting errors. It covers operator QEC, entanglement-assisted QEC, and continuous-time QEC. Part III (with contributions by Andrew Fletcher, Markus Grassl, Andreas Klappenecker, David Poulin, Martin Rötteler, and Mark Wilde) examines more advanced topics in quantum error-correcting codes,

and presents several important classes of specialized quantum codes, in particular quantum convolutional codes, nonadditive quantum codes, iterative quantum coding systems (along with a discussion of optimal decoding strategies), algebraic quantum coding theory, and optimization-based QEC. Part IV (with contributions by Ren-Bao Liu, Martin Rötteler, Zhen-Yu Wang, and Pawel Wocjan) considers advanced topics in dynamical decoupling, focusing in particular on high-order methods that allow error cancellation to any desired order in perturbation theory, and on the efficient construction of dynamical decoupling sequences using combinatorial methods. Part V (with contributions by Todd Brun, Debbie Leung, Daniel Lidar, Ognian Oreshkov, and Paolo Zanardi) considers alternatives to the standard “circuit model” of quantum computation, in particular holonomic and measurement-based quantum computation. Part VI (with contributions by Héctor Bombín, Austin Fowler, and Kovid Goyal) concerns topological methods in QEC, particularly topological codes and cluster state quantum computation. Part VII (with contributions by Dave Bacon, Jacob Taylor, Lorenza Viola, and Mark Wilde) deals with issues of practical implementation. It covers representative early experiments in QEC and dynamical decoupling, and discusses the implementation of error correction and fault-tolerance ideas in quantum computing architectures and quantum communication. Finally, Part VIII (with contributions by Robert Alicki, Harold Baranger, Eduardo Mucciolo, and Eduardo Novais) takes a critical look at the assumptions going into the fault-tolerant threshold theorems, and how they may not necessarily hold in realistic systems.

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