

Collective Preference and Choice

Collective decision making is a familiar feature of our social, political, and economic lives. It ranges from the relatively trivial (e.g. the choice of the next family car) to the globally significant (e.g. whether or not a country should go to war). Yet, whether trivial or globally significant, such decisions involve a number of challenging problems. These problems arise in the standard social choice setting, where individuals differ in their preferences. They also arise in the standard decision-making setting, where individuals share the same preferences, but differ in their decisional capabilities. The distinctive feature of *Collective Preference and Choice* is that it looks at classical aggregation problems that arise in three closely related areas: social choice theory, voting theory, and group decision making under uncertainty. Using a series of exercises and examples, the book explains these problems with reference to a number of important contributions to the study of collective decision making.

SHMUEL NITZAN is Sir Issac Wolfson Professor of Economics at Bar-Ilan University, Israel. He is the author (with Jacob Paroush) of *Collective Decision Making* (1986) and (with Gil Epstein) of *Endogenous Public Policy and Contests* (2007). His research focuses on political economy, social choice, collective decision making, public economics, and voting theory.



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SHMUEL NITZAN





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Dedicated to the memory of my mother and father



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Preface

The study of *collective preference and choice* involves exposure to a number of interesting and challenging problems. Clarifying these problems and their significance is the main objective of this book.

The problems of collective decision making arise in the standard social choice setting, where individuals differ in their preferences. They also occur in the standard team or committee decision-making setting, where individuals share the same preferences, but differ in their decisional capabilities in an uncertain environment. The issues of collective preference and choice are relevant and of significance to many disciplines in the social sciences.

The term "social preference" relates to the description of the system of tastes or preferences of society. The term "social choice" relates to behavior; that is, the selection of one of the feasible actions or alternatives faced by society. The book presents the classical aggregation problems of (i) social choice theory, (ii) voting theory and (iii) group decision making under uncertainty, and discusses their implications.

The social choice problem is clarified by Arrow's impossibility theorem, Sen's paretian liberal paradox and Gibbard–Satterthwaite impossibility theorem. The basic problems of voting theory are examined by focusing on Condorcet's voting paradox and on the comparison between the voting rules proposed by Condorcet, Dodgson and Borda. The problem of judgment aggregation is studied using the framework of Condorcet's jury theorem.

The book also presents three more recent approaches to the resolution of the problems in these three fields. These are, respectively, the metric approach to social compromise, the attempt to ameliorate



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majority decisiveness (tyranny) by using scoring rules, and the identification of the optimal group decision rule in the context of uncertain dichotomous choice.

There are relatively few textbooks focusing on social choice, on voting theory or on collective decision making under uncertainty. To the best of my knowledge, none of these relates to the main problems in **all** three (related) fields. This book provides such coverage, while not attempting to treat the three themes comprehensively.

The book has twelve chapters. The first part of the book (Chapters 1–3) has three objectives: to explain the basic reasons for the problematic nature of preference and decision aggregation (Chapter 1); to provide a brief overview of the problems presented in the book (Chapter 2); and to clarify the relationship between preferences and choice (Chapter 3). This relationship is relevant in discussions of both individual and social preferences and choices.

The second part of the book (Chapters 4–10) deals with individuals – voters, decision makers – who have different preferences. In this part, we discuss the main problems of social choice under non-strategic behavior (Chapters 4, 5 and 6), and some basic problems of voting theory (Chapters 7 and 8). The last two chapters in this part, Chapters 9 and 10, deal with strategic behavior.

The third part of the book (Chapters 11-12) deals with individuals who share the same preferences, but differ in their decisional skills. The analysis in this part uses the uncertain dichotomous choice model.

Unlike in other areas of economic theory and political thought, no book has to date presented issues of collective preferences and choice in a simple, relatively brief and yet profound way, accessible even to those who are not close to the field. The reason for this is twofold. First, the treatment of the topic in the literature has been based on intensive use of mathematical tools and consequently the presentation of the material to a relatively broad readership in "soft" non-technical language is difficult. Second, the topic is interdisciplinary and therefore tends to "fall between the cracks."



PREFACE XV

The study of the problems of collective preferences and choice has gathered momentum in the past fifty years, though some of the main problems were already being discussed in the eighteenth and nineteenth centuries. The findings obtained began to play an important role in sub-disciplines such as Political Theory, Public Economics and Welfare Economics, and contributed to the development of new sub-disciplines such as Social Choice, Public Choice, Voting Theory and Collective Decision Making. A noteworthy indication of increased recognition in the significance of the subject is the fact that since 1972, six of the recipients of the Nobel prize in Economics – Paul Samuelson, Kenneth Arrow, John Nash, John Harsanyi, Amartya Sen, and William Vickrey – achieved their high recognition partly because of their contribution to the clarification of the problematic nature of social choice, or their contribution to the resolution of some of the problems associated with collective decision making.

In summary, this book does not aim to systematically and comprehensively cover the numerous topics related to collective preferences and choice. Its objective is to clarify the main problems of collective decision making and arouse the interest of the reader to further and deeper study of the fascinating problems of social choice theory, voting theory and group decision making under uncertainty.

Finally, the presentation of the material in this book has been influenced by my considerable experience of teaching these topics. In particular, I have included a relatively large number of problems together with their detailed answers. I hope this will help readers and students to better understand the material, be able to apply it – and perhaps even enjoy it!



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