Modern Electrodynamics

An engaging writing style and a strong focus on the physics make this comprehensive, graduate-level textbook unique among existing classical electromagnetism textbooks.

Charged particles in vacuum and the electrodynamics of continuous media are given equal attention in discussions of electrostatics, magnetostatics, quasistatics, conservation laws, wave propagation, radiation, scattering, special relativity, and field theory. Extensive use of qualitative arguments similar to those used by working physicists makes Modern Electrodynamics a must-have for every student of this subject.

In 24 chapters, the textbook covers many more topics than can be presented in a typical two-semester course, making it easy for instructors to tailor courses to their specific needs. Close to 120 worked examples and 80 applications boxes help the reader build physical intuition and develop technical skill. Nearly 600 end-of-chapter homework problems encourage students to engage actively with the material. A solutions manual is available for instructors at www.cambridge.org/Zangwill.

Andrew Zangwill is a Professor of Physics at the Georgia Institute of Technology and a Fellow of the American Physical Society. He is the author of the popular monograph Physics at Surfaces (Cambridge University Press, 1988).
There are more things in heaven & earth connected with electromagnetism than are yet dreamt of in philosophy.

Joseph Henry, letter to Lewis C. Beck (1827)


The search for reason ends at the shore of the known; on the immense expanse beyond it only the ineffable can glide.

Abraham Joshua Heschel, Man is Not Alone (1951)

Why repeat all this? Because there are new generations born every day. Because there are great ideas developed in the history of man, and these ideas do not last unless they are passed purposely and clearly from generation to generation.

Richard Feynman, The Meaning of It All (1963)
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