CONTEMPORARY KINETIC THEORY OF MATTER

Kinetic theory provides a microscopic description of many observable, macroscopic processes and has a wide range of important applications in physics, astronomy, chemistry, and engineering. This powerful, theoretical framework allows a quantitative treatment of many nonequilibrium phenomena such as transport processes in classical and quantum fluids. This book describes in detail the Boltzmann equation theory, obtained in both traditional and modern ways. Applications and generalizations describing nonequilibrium processes in a variety of systems are also covered, including dilute and moderately dense gases, particles in random media, hard-sphere crystals, condensed Bose–Einstein gases, and granular materials. Fluctuation phenomena in nonequilibrium fluids and related non-analyticities in the hydrodynamic equations are also discussed in some detail. A thorough examination of many topics concerning time-dependent phenomena in material systems, this book describes both current knowledge as well as future directions of the field.

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To
Matthieu Ernst, in honor of our friendship and collaborations and
his many contributions to kinetic theory

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Nomenclature

$E(k)$	Energy of Bogoliubov excitations
$[\boldsymbol{D}^{(\alpha)}, \boldsymbol{D}^{(\gamma)}]$	Bracket integral
α	Accomodation coefficient for a boundary
α_T	Coefficient of thermal expansion
$\bar{\mathbf{T}}_0$	Binary collision operator for binary collisions, when the duration of the collision and the spatial separations of the colliding particles are ignored
$\bar{\mathbf{T}}_W$	Binary collision operator for wall–particle collisions
$\mathbf{T}_{\pm}(1,2)$	Hard-sphere binary collision operators
$\mathbf{T}^r_{\pm}(1,2)$	Real part of a hard-sphere binary collision operator
$\mathbf{T}^v_{\pm}(1,2)$	Virtual part of a hard-sphere binary collision operator
$\bar{\mathbf{T}}_{\pm}(1,2)$	Barred, or adjoint, of a hard-sphere binary collision operator, $T_{\mp}(1,2)$
$\bar{\mathcal{L}}_{0,W-}$	Free streaming part of Liouville operator including particle–wall interactions
$\bar{\mathcal{L}}^{(ps)}_{\pm}(N)$	Barred pseudo-Liouville operator for N hard spheres
$\bar{\mathcal{L}}^{(ps)}_{W-}(n)$	Pseudo-Liouville operator including particle-wall interactions
$\boldsymbol{b}_{\hat{k}}(1,2)$	Binary collision velocity exchange operator that replaces velocities by their restituting values
β	Inverse temperature parameter
$\boldsymbol{F}$	External force per unit mass in the Boltzmann equation
$\boldsymbol{F}_{ext}$	External force in Langevin equation
$\mathbf{k}$	Wave vector
$\boldsymbol{\rho}(t)$	Density matrix
Ω	Angular velocity vector
ρ_S	Location of a point on the boundary surface of a system
σ_{ij}	Elements of the stress tensor
$\mathbf{D}$	Velocity gradient tensor

$\mathbf{g}$	Relative velocity of two particles
$\mathbf{J_K}(\boldsymbol{r},t)$	Local energy current vector
$\mathbf{P}(\boldsymbol{r},t)$	Local pressure tensor
$\mathbf{q}(\boldsymbol{r},t)$	Local energy current
$\mathbf{u}(\mathbf{r},t)$	Local average velocity in a gas at point $\boldsymbol{r}$ at time t
$\mathbf{c}=\mathbf{v}-\mathbf{u}(\mathbf{r},t)$	Peculiar velocity of a particle
$\boldsymbol{\Lambda}(\mathbf{v})$	Boltzmann collision operator linearized about a total equilibrium distribution function
$\boldsymbol{\Lambda}_{\alpha\beta}$	Linearized Boltzmann collision operator for binary collisions of particles of species α and β
$\boldsymbol{\Lambda}_{loc}(\mathbf{v_1})$	Boltzmann collision operator linearized about a local equilibrium distribution
$\boldsymbol{V}$	N-particle $2dN$-dimensional velocity vector in phase space
$\boldsymbol{u}$	Scaled velocity for granular gas
$\boldsymbol{u}_n$	Velocity of the normal fluid in a condensed boson gas
χ_T	Isothermal compressibility
ℓ	Mean free path length
ϵ_n	Coefficient of restitution – normal
ϵ_t	Coefficient of restitution – tangential
ϵ	The azimuthal angle
η	Coefficient of shear viscosity
$\eta(\boldsymbol{r},t)$	External source in the η-ensemble
η_E	Enskog theory coefficient of shear viscosity
$\eta_i(t),\epsilon_i(t)$	Descriptors for presence or absence of white or black beads at point i at time t in the Kac ring model
γ	Drag coefficient in Langevin equation
$\Gamma_+ d\mathbf{r}d\mathbf{v}$	Rate at which the number of particles with prescribed velocities increase due to binary collisions in a very small $2d$-dimensional one-particle position and velocity phase space
$\Gamma_- d\mathbf{r}d\mathbf{v}$	Rate at which the number of particles with prescribed velocities decrease due to binary collisions in a very small $2d$-dimensional one-particle position and velocity phase space
Γ_s	Sound damping coefficient
$\Gamma_{S,E}$	Enskog theory value of the sound damping coefficient
Γ_s	Parameter descibing the cooling rate in a granular gas
$\Gamma_W d\mathbf{r}d\mathbf{v}$	Rate of change of the single particle distribution function due to collisions of particles with a boundary wall in a small $2d$-dimensional, one-particle phase space

$\hat{\mathbf{k}}$	Unit vector in the direction of the vector from the origin to the point of closest approach in binary collision as described in the relative coordinate system centered on one of the colliding particles
$\hat{\mathbf{V}}$	Particle–particle Interaction contribution to the Hamiltonian operator
$\hat{\sigma}$	Unit vector along apse line for hard-sphere collisions
λ	Coefficient of thermal conductivity
λ'	Partial coefficient of thermal conductivity
$\lambda/(\eta c_v)$	Eucken factor
λ_E	Enskog theory value of the coefficient of thermal conductivity
$\Lambda_i^{(\pm)}(\boldsymbol{r}, \boldsymbol{p}, t)$	Positive and negative stretching factors
$\mathcal{L}_{\mathbf{k}}$	Linear Boltzmann propagator acting on deviation of the single particle distribution from its equilibrium value, χ
$\mathcal{L}_{\mathbf{k}}^{(R)}(z)$	Linear single particle ring propagator
$\mathcal{W}_N(x_1, x_2, \ldots, x_N)$	General N-particle function of positions and momenta symmetric under particle interchanges
$\mathcal{G}_0(1, 2, \ldots, s, z)$	Laplace transform of time displacement operator; also called a propagator
$\mathcal{L}(\Gamma)$	N-particle Liouville operator
$\mathcal{L}_0(\Gamma)$	Kinetic part of the N-particle Liouville operator
$\mathcal{L}_{\pm}^{(ps)}(N)$	Pseudo-Liouville operator for N hard spheres
$\mathcal{L}_I(\Gamma)$	Interaction potential part of the N-particle Liouville operator
$\mathcal{S}(\Gamma)$	Time displacement operator in phase space
$\mathcal{S}_t^{(0)}(1, 2, \ldots, s)$	s-particle free streaming operator
$\mathcal{V}_s^{(eq)}$	Husimi cluster functions for s-particles
μ	Ordering parameter in the Chapman-Enskog solution of the Boltzmann equation
μ_{12}	Reduced mass of two particles
ν	Collision frequency parameter in Bhatnagar–Gross–Krook (BGK) model
$\nu(v_i)$	Low-density, equilibrium collision frequency for a particle with velocity $\mathbf{v}_i$
ν_c	Collision frequency
ω	Thermal creep coefficient
$\omega^{(\pm)}$	Leading order term in sound mode eigenvalue
$\omega_i(k)$	Hydrodynamic eigenvalues
Ω_{ij}	Non-dissipative terms in the matrix form of the linearized Navier–Stokes equations
$\mathcal{P}, \mathcal{P}_{\perp}$	Zwanzig–Mori projection operators