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Groups, Graphs and Trees

An Introduction to the Geometry of Infinite Groups

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For my driver, Piotr

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Preface

Groups are algebraic objects, consisting of a set with a binary operation that satisfies a short list of required properties: the binary operation must be associative; there is an identity element; and every element has an inverse. Presenting groups in this formal, abstract algebraic manner is both useful and powerful. Yet it avoids a wonderful geometric perspective on group theory that is also useful and powerful, particularly in the study of infinite groups. This perspective is hinted at in the combinatorial approach to finite groups that is often seen in a first course in abstract algebra. It is my intention to bring the geometric perspective forward, to establish some elementary results that indicate the utility of this perspective, and to highlight some interesting examples of particular infinite groups along the way. My own bias is that these groups are just as interesting as the theorems.

The topics covered in this book fit inside of "geometric group theory," a field that sits in the impressively large intersection of abstract algebra, geometry, topology, formal language theory, and many other fields. I hope that this book will provide an introduction to geometric group theory at a broadly accessible level, requiring nothing more than a single-semester exposure to groups and a naive familiarity with the combinatorial theory of graphs.

The chapters alternate between those devoted to general techniques and theorems (odd numbers) and brief chapters introducing some of the standard examples of infinite groups (even numbers). Chapter 2 presents a few groups generated by reflections; Chapter 4 presents the Baumslag–Solitar group $BS(1,2)$ in terms of linear functions; Chapter 6 is the Gupta–Sidki variant of Grigorchuk's group; the Lamplighter group is discussed in Chapter 8; and Thompson's group F is the subject of Chapter 10. When I taught this material at Lafayette College I referred to the material in these even numbered chapters as "field trips to the Zoo of Infinite Groups."

The first chapter should be relatively easy to work through, as it reviews material on groups (mainly finite groups), group actions, and the combinatorial theory of graphs. It establishes quite a bit of notation and introduces the construction of Cayley graphs. While some material in this chapter may be new to the reader, most of it should seem to be a repackaging of ideas that she or he has previously encountered.

Chapter 3 is an introduction to free groups and free products of groups. Chapters 5 and 7 are devoted to connections between finitely generated groups and formal language theory. Chapters 9 and 11 deal with the geometry of infinite groups, with Chapter 9 focusing on what might be called the "fine geometry" of Cayley graphs, while Chapter 11 treats what is called the large-scale geometry of groups.

While no background beyond elementary group theory is necessary for this book, a broader undergraduate exposure to mathematics is certainly helpful. My experience in the classroom indicates that the material in Chapter 7 is demanding for people who have not previously encountered formal language theory. Similarly Chapter 11 is easier for people who have had a course in real analysis. Because hyperbolic geometry is not a standard undergraduate topic, Gromov's theory of hyperbolic groups does not appear in this book. Similarly, because algebraic topology is not a standard undergraduate topic, I have avoided fundamental groups and covering spaces.

There are two forms of exercises in this book. A few exercises are embedded within the chapters. These should be done, at least at the level of the reader convincing themselves they know how to do them, while reading through the material. There are also end-of-chapter exercises that are arranged in the order that material is presented in the chapter. Some of these end-of-chapter exercises are challenging but most are reasonably accessible.

Groups, Graphs and Trees was developed from notes used in two undergraduate course offerings at Lafayette College, and it can certainly serve as a primary text for an advanced undergraduate course. It should also be useful as a text for a reading course and as a gentle introduction to geometric group theory for mathematicians with a broader background than this. An undergraduate course that attempted to cover this text, omitting no details, in one semester, would have to move at a rather brisk pace. The critical background information is contained in the first five chapters and those should not be trimmed. With a bit of forethought an instructor can cover much of the rest of this book, if for example the material in Chapter 7 or Chapter 11 is presented more as a colloquium than as course material. My own hope is that various classes will find the space in their semester to pursue tangents of interest to them, and then let me know the results of their exploration.

I have many people to thank. My wife Trisha and son Robert were unreasonably supportive of this project. Many students provided important feedback as I fumbled through the process of presenting this bit of advanced mathematics at an elementary level: George Armagh, Kari Barkley, Jenna Bratz, Jacob Carson, Joellen Cope, Joe Dudek, Josh Goldstein, Ekaterina Jager, Brian Kronenthal, Rob McEwen, and Zachary Reiter. I also benefited from extensive feedback given by my colleagues Ethan Berkove and Jon McCammond. Finally, a number of anonymous referees provided comments on various draft chapters. I was impressed by the fact that there was no intersection between the comments provided by students, the comments provided by colleagues, and the anonymous referees!