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Preface

The first version of these lecture notes was composed for a last-year undergraduate course at Chalmers University of Technology, in the spring semester 2000. I wrote a revised and expanded version for the same course one year later. This is the third and final (?) version.

The notes are intended to be sufficiently self-contained that they can be read without any supplementary material, by anyone who has previously taken (and passed) some basic course in probability or mathematical statistics, plus some introductory course in computer programming.

The core material falls naturally into two parts: Chapters 2–6 on the basic theory of Markov chains, and Chapters 7–13 on applications to a number of randomized algorithms.

Markov chains are a class of random processes exhibiting a certain "memoryless property", and the study of these – sometimes referred to as Markov theory – is one of the main areas in modern probability theory. This area cannot be avoided by a student aiming at learning how to design and implement randomized algorithms, because Markov chains are a fundamental ingredient in the study of such algorithms. In fact, any randomized algorithm can (often fruitfully) be viewed as a Markov chain.

I have chosen to restrict the discussion to discrete time Markov chains with finite state space. One reason for doing so is that several of the most important ideas and concepts in Markov theory arise already in this setting; these ideas are more digestible when they are not obscured by the additional technicalities arising from continuous time and more general state spaces. It can also be argued that the setting with discrete time and finite state space is the most natural when the ultimate goal is to construct algorithms: Discrete time is natural because computer programs operate in discrete steps. Finite state space is natural because of the mere fact that a computer has a finite amount of memory, and therefore can only be in a finite number of distinct

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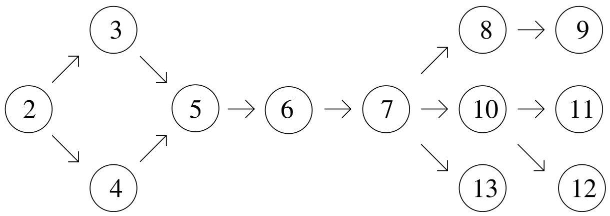
"states". Hence, the Markov chain corresponding to a randomized algorithm implemented on a real computer has finite state space.

However, I do not claim that more general Markov chains are irrelevant to the study of randomized algorithms. For instance, an infinite state space is sometimes useful as an approximation to (and easier to analyze than) a finite but very large state space. For students wishing to dig into the more general Markov theory, the final chapter provides several suggestions for further reading.

Randomized algorithms are simply algorithms that make use of random number generators. In Chapters 7–13, the Markov theory developed in previous chapters is applied to some specific randomized algorithms. The Markov chain Monte Carlo (MCMC) method, studied in Chapters 7 and 8, is a class of algorithms which provides one of the currently most popular methods for simulating complicated stochastic systems. In Chapter 9, MCMC is applied to the problem of counting the number of objects in a complicated combinatorial set. Then, in Chapters 10–12, we study a recent improvement of standard MCMC, known as the Propp–Wilson algorithm. Finally, Chapter 13 deals with simulated annealing, which is a widely used randomized algorithm for various optimization problems.

It should be noted that the set of algorithms studied in Chapters 7–13 constitutes only a small (and not particularly representative) fraction of all randomized algorithms. For a broader view of the wide variety of applications of randomization in algorithms, consult some of the suggestions for further reading in Chapter 14.

The following diagram shows the structure of (essential) interdependence between Chapters 2–13.



How the chapters depend on each other.

Regarding exercises: Most chapters end with a number of problems. These are of greatly varying difficulty. To guide the student in the choice of problems to work on, and the amount of time to invest into solving the problems, each problem has been equipped with a parenthesized number between (1) and

`(10)` to rank the approximate size and difficulty of the problem. `(1)` means that the problem amounts simply to checking some definition in the chapter (or something similar), and should be doable in a couple of minutes. At the other end of the scale, `(10)` means that the problem requires a deep understanding of the material presented in the chapter, and at least several hours of work. Some of the problems require a bit of programming; this is indicated by an asterisk, as in `(7*)`.

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