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An Introduction to Lie Groups and Lie Algebras

With roots in the nineteenth century, Lie theory has since found many and varied applications in mathematics and mathematical physics, to the point where it is now regarded as a classical branch of mathematics in its own right. This graduate text focuses on the study of semisimple Lie algebras, developing the necessary theory along the way.

The material covered ranges from basic definitions of Lie groups, to the theory of root systems, and classification of finite-dimensional representations of semisimple Lie algebras. Written in an informal style, this is a contemporary introduction to the subject which emphasizes the main concepts of the proofs and outlines the necessary technical details, allowing the material to be conveyed concisely.

Based on a lecture course given by the author at the State University of New York at Stony Brook, the book includes numerous exercises and worked examples and is ideal for graduate courses on Lie groups and Lie algebras.

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An Introduction to Lie Groups and Lie Algebras

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Dedicated to my teachers

Contents

Preface

This book is an introduction to the theory of Lie groups and Lie algebras, with emphasis on the theory of semisimple Lie algebras. It can serve as a basis for a two-semester graduate course or – omitting some material – as a basis for a rather intensive one-semester course. The book includes a large number of exercises.

The material covered in the book ranges from basic definitions of Lie groups to the theory of root systems and highest weight representations of semisimple Lie algebras; however, to keep book size small, the structure theory of semisimple and compact Lie groups is not covered.

Exposition follows the style of famous Serre's textbook on Lie algebras [47]: we tried to make the book more readable by stressing ideas of the proofs rather than technical details. In many cases, details of the proofs are given in exercises (always providing sufficient hints so that good students should have no difficulty completing the proof). In some cases, technical proofs are omitted altogether; for example, we do not give proofs of Engel's or Poincare–Birkhoff–Witt theorems, instead providing an outline of the proof. Of course, in such cases we give references to books containing full proofs.

It is assumed that the reader is familiar with basics of topology and differential geometry (manifolds, vector fields, differential forms, fundamental groups, covering spaces) and basic algebra (rings, modules). Some parts of the book require knowledge of basic homological algebra (short and long exact sequences, Ext spaces).

Errata for this book are available on the book web page at
`http://www.math.sunysb.edu/~kirillov/liegroups/`.