

Cambridge University Press  
978-0-521-88944-5 - Gravity's Fatal Attraction: Black Holes in the Universe, Second Edition  
Mitchell Begelman and Martin Rees  
Excerpt  
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# 1 GRAVITY TRIUMPHANT

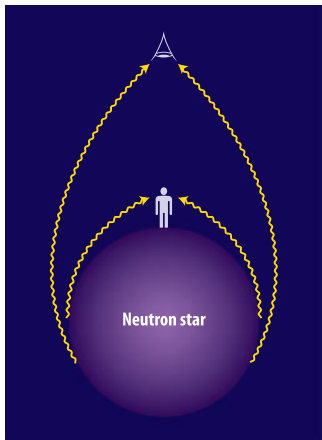
## Chapter 1

Gravity is the one truly universal force. No substance, no kind of particle, not even light itself, is free of its grasp. It was Isaac Newton who, over three hundred years ago, realized that the force holding us to the ground, and governing a cannon ball's trajectory, is the same force that holds the Moon in its orbit around the Earth. This force, which later came to be called gravity, is the mutual attraction among all bodies. Newton showed how the motion of each planet in the Solar System is the combined outcome of the gravitational pull of the Sun and all the other planets, each contributing according to its mass and distance from the others. Out of Newton's conceptually simple prescriptions came the calculations that have guided spacecraft to all the planets, told us a year in advance that Comet Shoemaker–Levy would hit Jupiter, and enabled us to determine the mass of the Milky Way Galaxy. On cosmic scales gravity dominates over every other kind of force. Every significant level of structure in the Universe – stars, clusters of stars, galaxies, and clusters of galaxies – is maintained by the force of gravity.

Nowhere is gravity stronger than near the objects we call black holes. The effect of gravity at the surface of a body intensifies if the mass of the body is increased, or its size is decreased. Gravity's strength can be characterized by how fast a rocket must be fired to escape from the body. For the Earth, the escape speed is 40 000 kilometers per hour; for the Sun it is nearly a hundred times greater. This tendency for the escape speed to become ever larger for more massive, or more compact, bodies raises an obvious conceptual problem. The gravitational pull of the Sun dominates the Solar System, yet its escape speed is only 1/500 the highest speed possible, the speed of light. What would be the effect of gravity around a body for which the escape speed was as high as that of light?

The Reverend John Michell, an underappreciated polymath of eighteenth-century science, puzzled over this question in 1784. Arguing on the basis of Newton's laws and assuming that light behaves like particles of matter (the view favored by Newton and his contemporaries), Michell wrote a paper in which he noted, "If the semi-diameter of a sphere of the same density as the Sun were to exceed that of the Sun in the proportion of five hundred to one, and supposing light to be attracted by the same force in proportion to its *vis inertiae* [inertial mass] with other bodies, all light emitted from such a body would be made to return towards it, by its own proper gravity." With remarkable foresight, Michell was suggesting, as did the French astronomer and mathematician Pierre-Simon Laplace a decade later, that the most massive objects in the Universe might be undetectable by their direct radiation but still manifest gravitational effects on material near them. Nearly two hundred years later, astronomers discovered objects for which gravity is as strong as Michell envisaged – these powerfully gravitating bodies are neutron stars and black holes.

General Relativity and Black Holes



**The severe bending of light by gravity means that an observer on the surface of a neutron star can see farther to the horizon. An observer high above the star could see much more than half the surface.**

Neutron stars, discovered in 1968, are objects slightly heavier than the Sun but extending only 10 to 20 kilometers across, with an escape speed about half that of light. While light can escape from a neutron star, its path is strongly curved by gravity, just like the trajectory of a rocket boosting a spacecraft into interplanetary space from the Earth. The extreme strength of gravity around a neutron star leads to consequences far removed from common-sense experience. Because light itself follows a strongly curved trajectory, an observer’s perspective would be severely distorted near a neutron star. One would be able to see much farther to the horizon, and from high above the surface, more than half the entire star would be visible. So powerful is gravity that a pen dropped from a height of one meter above a neutron star would release as much energy as a ton of dynamite.

Conditions near a neutron star are not quite as extreme as Michell described for his hypothetical light-retaining body. But if a neutron star were a few times smaller, or just slightly heavier, it would trap all the light in its vicinity. It would then be a black hole. Black holes are not merely stars whose surface is hidden from view by the bending of light. In 1905, Einstein’s special theory of relativity established that the speed of light sets a limit to the speed at which any kind of matter or signal propagates, so that the kind of object envisaged by Michell would be completely cut off from the outside Universe. Nothing that ventures inside it would ever be able to escape. As it turns out, in a black hole gravity so overwhelms other forces that matter is crushed virtually to a point. These bizarre and counterintuitive properties led many researchers to suppose that, somehow, matter could never become quite so condensed that it produced a black hole. But we now know that black holes are not mere theoretical constructs; they exist in profusion and account for many of the most spectacular astronomical discoveries of recent times.

This book describes how black holes were found and what their existence implies for cosmic evolution and our understanding of gravity. The first candidate black holes were discovered soon after neutron stars. Since then, evidence has mounted that millions of black holes exist in our Milky Way Galaxy, each with a mass roughly 10 times that of the Sun. And in the centers of most galaxies, there almost certainly exist monster black holes weighing as much as 100 million suns, similar in mass and radius to the objects conjectured by Michell.

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Neither Newtonian gravity nor Newton’s laws of motion are applicable to situations where the gravitational fields are as strong as those conjectured by Michell and Laplace. On the Earth, as elsewhere in the Solar System,

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Newton's theory of gravitation affords an excellent approximation. But where gravity is so strong that it can accelerate matter to nearly the speed of light – approximately 300 000 kilometers per second – the Newtonian description no longer works. We cannot formulate a consistent picture of a black hole without deeper insights into gravity than Newton's theory offers. Fortunately, long before black holes were discovered, Einstein's general theory of relativity had provided such insights and laid a firm theoretical foundation for our understanding of black holes. Einstein's theory has been verified by increasingly precise experimental tests, and its equations have solutions that represent situations where gravity is overwhelmingly strong, as it is in black holes.

The papers that established Albert Einstein as the greatest physicist since Newton are all contained in just two volumes of the journal *Annalen der Physik*, published in the years 1905 and 1916. The old backnumbers of scientific journals, relegated to remote stock-rooms in academic libraries, are usually seldom consulted, but these two particular volumes are rare and valued collector's items. In 1905, the 26-year-old Einstein enunciated his "special theory of relativity," which (among other breakthroughs) emphasized the special significance of the speed of light and postulated the fundamental interconvertability or "equivalence" between mass and energy popularly paraphrased in the expression  $E = mc^2$ . He also proposed that light is quantized into "packets" of energy and formulated the statistical theory of Brownian motion, the frenetic dance of dust particles suspended in a liquid or gas. These contributions alone would cause Einstein to be ranked among the half-dozen great pioneers of twentieth-century physics.



**Albert Einstein in 1905, aged 26. In that year he proposed the special theory of relativity, deduced the quantum nature of light, and explained Brownian motion of dust particles.**

But it is his gravitational theory, "general" relativity, developed eleven years later, that puts Einstein in a class by himself. Had he written none of his 1905 papers, it would not have been long before the same concepts would have been put forward by some of his distinguished contemporaries: the ideas were "in the air." Well-known inconsistencies in earlier theories and puzzling experimental results would, in any case, have focused interest on these problems. General relativity, however, was not a response to any particular observational enigma. Motivated by more than the desire to explain concrete observations, Einstein sought simplicity and unity. Whereas Newton conceived gravity as a force instantaneously transmitted between bodies – a view clearly inconsistent with the speed limit on the propagation of signals – Einstein postulated that gravitational fields are manifestations of the curvature of space itself. Masses do not "exert" a gravitational pull, deflecting bodies from a straight path. Rather, their presence distorts the space within and around them. According to general relativity, bodies moving through space follow the straightest path possible through an amalgam of space and time called "spacetime." But when space is distorted, these paths become curved and accelerating

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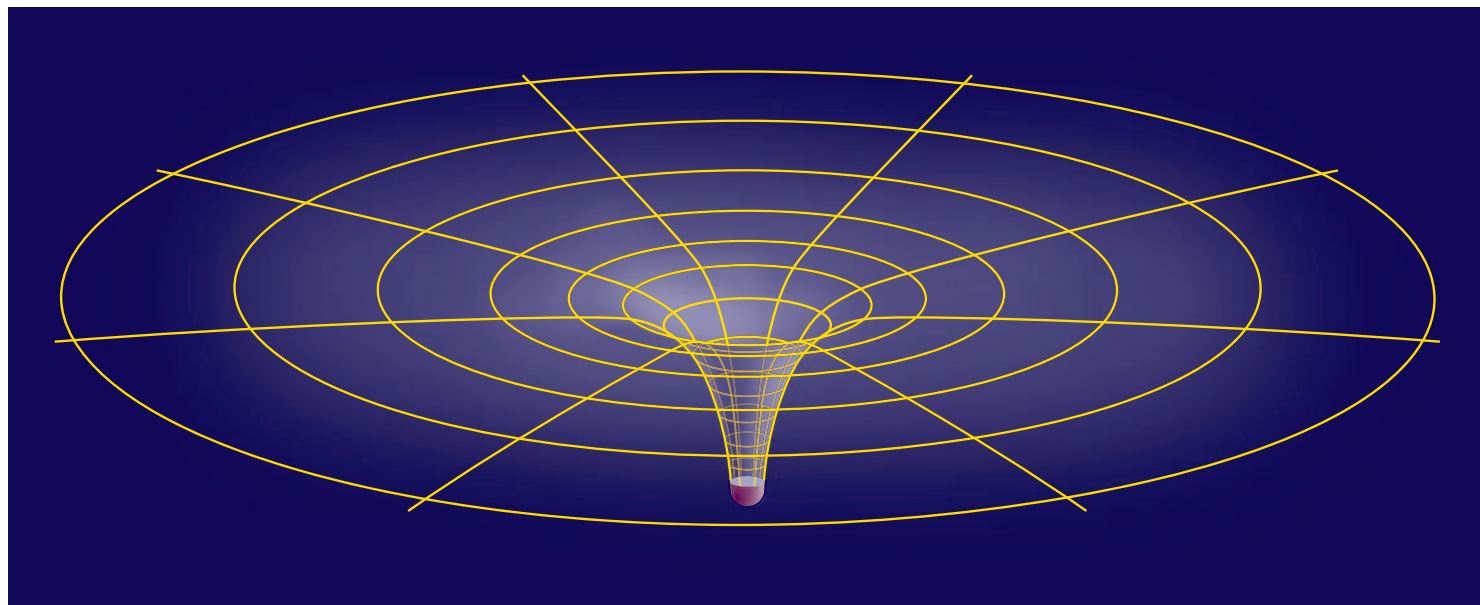
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## General Relativity and Black Holes



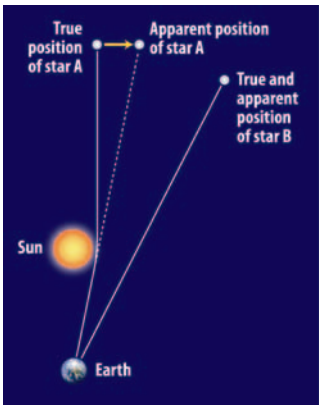
**According to the general theory of relativity, a gravitating body stretches and distorts the space around it, much like a lead weight resting on a rubber membrane. Particles following the straightest possible paths in curved spacetime appear to be responding to an attractive force. If the gravitating object represented were a black hole, the depression would be infinitely deep.**

trajectories that we might interpret as the reaction to a force. In the words of the noted relativist John Archibald Wheeler, “Space tells matter how to move; matter tells space how to curve.”

As Einstein himself said when he announced his new work, “Scarcely anyone who has fully understood the theory can escape from its magic.” The physicist Hermann Weyl described it as “the greatest example of the power of speculative thought,” and Max Born called it “the greatest feat of human thinking about Nature.” Had it not been for Einstein, an equally comprehensive theory of gravity might not have arrived until decades later, and might well have been approached by quite a different route. Einstein’s creativity thus put a uniquely individual and long-lasting imprint on modern physics. Most scientists hope that their work will last, but Einstein is unique in the degree to which his work retains its individual identity.

Indeed, general relativity was proposed so far in advance of any real application that it remained, for forty years after its discovery, an austere intellectual monument – a somewhat sterile topic isolated from the mainstream of physics and astronomy – whose practitioners, according to Thomas Gold, were “magnificent cultural ornaments.” This remoteness from the mainstream stands in glaring contrast to its more recent status as one of the liveliest frontiers of fundamental research. Its reputation for being an exceptionally hard subject was sometimes exaggerated, not least by its early practitioners. (When Arthur Eddington was asked if it were true that just three people in the world understood general relativity, he

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During the 1919 solar eclipse, Arthur Eddington observed the deflection of starlight by the Sun's gravity. The effect is tiny; according to Einstein's theory the maximum angle of deflection would be 1.75 seconds of arc (1/2000 of a degree) for a light ray grazing the Sun's surface.

is rumored to have wondered who the third might be.) The theory is now taught almost routinely to physics students, along with quantum physics, electromagnetism, and the rest of the physics canon.

To be sure, general relativity also had its share of early experimental successes. It accounted for long-recognized anomalies in Mercury's orbit, and was famously confirmed by the results of the 1919 solar eclipse expedition led by the British astrophysicist Arthur Eddington. Eddington's observations revealed that light rays passing close to the Sun were deflected by just the amount predicted by Einstein's theory. (A Newtonian argument, as might have been used by Michell, would have predicted half the observed amount.) For slowly moving bodies in weak gravitational fields, the predictions of general relativity agree almost exactly with those of Newton's laws – as they must, given that the latter have withstood three hundred years of testing. In the Solar System, the predicted departures from Newtonian theory amount to only about one part in a million. But experiments, including the tracking of space probes and higher-precision versions of Eddington's light-deflection measurements, are now precise enough to determine these tiny differences within a better than 1 percent margin of error. General relativity has been verified in every single case and, moreover, has proved more accurate than other, similar theories.

The recent revival of interest in gravitational physics has been inspired not only by the high-precision measurements whose results have favored Einstein's theory over its rivals, but also – to a greater extent – by dramatic advances in astronomy and space research. In neutron stars and black holes, the distinctive features of general relativity take center stage; they are not merely trifling modifications to Newtonian theory. These objects offer us the opportunity to test Einstein's theories in new ways. The other arena where relativity is crucial is cosmology – the study of the entire expanding Universe. During the past forty years, cosmology has evolved from a subject about which we knew little more than Hubble's law of universal expansion (which states that distant galaxies appear to recede from us at a speed that is proportional to their distance) to a science that allows us to infer cosmic history reliably back to one second after the big bang, and to make compelling conjectures about still earlier eras. The entire fate of the Universe depends on the relativistic curvature of space induced by everything in it. And black holes may have played a special role in the evolution of the Universe, as we shall show in late chapters.

From a relativistic point of view, the strength of a gravitational field can be characterized by the ratio of the velocity required to escape from the field (the "escape speed" often discussed in the context of space exploration) to the speed of light, all squared. When this ratio is very small, as it is near the Sun (1/250 000), Newtonian theory provides a near-perfect



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description of gravity. But when this ratio approaches 1, as it does close to a neutron star or black hole, general relativity predicts very bizarre and dramatic effects. We have already mentioned the bending of light rays, which becomes extreme as they traverse the curved space associated with a strong gravitational field. Perhaps even stranger is the “time dilation” effect – the apparent slowing down of time in the presence of gravity. If space were marked out by a set of fixed clocks, those located where gravity is strong would seem to a distant observer (located where gravity is weak) to run slow; conversely, distant clocks would seem to run fast when viewed by observers near the central mass. No matter how much the rate at which time passes varies from place to place, at a given spot time would pass at the same rate for *every* process or event. If we could send astronauts to the surface of a neutron star, and listen to them through a radio or view them through a telescope, their speech and movements would appear to us to be about 25 percent slower than normal. As far as they were concerned, however, they would be operating at normal speed and our messages would seem speeded up. Any oscillation of the kind that generates an electromagnetic wave would behave just like the ticks of a clock. To a distant observer watching such an oscillation near the surface of a large mass, the interval from one oscillation to the next would appear longer, and so would the wavelength of the corresponding electromagnetic wave. Because in the optical spectrum red light is longer in wavelength than the other colors, the lengthening of wavelength is called a redshift. Thus light emitted close to the central mass would display a “gravitational redshift” when observed farther out; in other words, its observed wavelength is longer than the wavelength when it was emitted.



**Karl Schwarzschild in his academic robe of the University of Göttingen, Germany. By the winter of 1915–16, he had voluntarily taken leave from his position as director of Potsdam Astrophysical Observatory for active duty in World War I. He was dying of a rare disease contracted on the eastern front when he discovered the Schwarzschild radius.**

Black holes exhibit the effects of “strong” gravity in its purest, and most extreme, form. The possible existence of objects that have collapsed to such small dimensions that neither light nor any other signal can escape from them is predicted by most theories of gravity, not merely by Einstein’s general relativity. But it is within the context of general relativity that the most complete understanding of black holes and their properties has been developed.

The first theoretical description of a black hole within the framework of general relativity was given in 1916 by Karl Schwarzschild, who calculated the distortions of space outside a spherically symmetrical body of a given mass. An observer, sitting at a fixed distance from the center of the sphere, would measure the same curvature of space (and therefore the same gravitational strength) regardless of the body’s radius. However, if the body were smaller than a certain size, it would not be seen, even though its gravity would still be felt. This boundary between visibility and invisibility, or “horizon,” corresponds to the minimum radius from which light or anything else can escape to an external observer, or, almost equivalently, the radius at which

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the gravitational redshift is infinite and the escape speed is equal to the speed of light. Objects (and light) can cross the horizon in the inward direction, but nothing can come out. Any spherical body smaller than this radius is a “Schwarzschild black hole.” (Schwarzschild’s equations are discussed in the box below.)

The horizon of a Schwarzschild black hole lies at a radius of  $3M$  kilometers, where  $M$  is the mass of the black hole measured in solar masses (units of mass each equivalent to the mass of our Sun). Coincidentally, this is exactly the same radius that Michell and Laplace would have calculated using the incorrect Newtonian theory.

The Schwarzschild Metric and the Curvature of Space

According to Newton, the gravitational force outside a spherical object is inversely proportional to the square of the distance from the object’s center. From this finding, it follows that orbits around such an object must be ellipses. In 1916, Karl Schwarzschild determined how gravity would behave outside a spherical object according to Einstein’s theory. The results are expressed as the *Schwarzschild metric*, describing the curvature of space outside a spherically symmetric mass.

Where gravity is weak and the orbital motion is very much slower than the speed of light, the predictions of Schwarzschild’s metric agree almost exactly with Newton’s. The Earth moves around the Sun at about one ten-thousandth the speed of light, and Newton’s theory is an excellent approximation for all orbits in the Solar System. According to general relativity, however, orbits around a central mass don’t close up as exact ellipses, as Newton’s theory predicts. Whereas Newtonian theory predicts that the orientation of an elliptical orbit remains fixed in space, Einstein’s theory predicts that the direction of the “long axis” of the ellipse gradually rotates. The expected

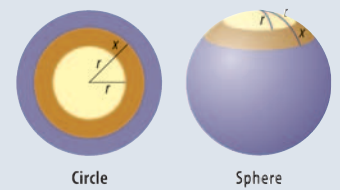
effect has been detected in Mercury’s orbit (though the “Newtonian” perturbations due to the other planets are much stronger). Another consequence of the Schwarzschild metric is that light moving through the Solar System is deflected twice as much as it would be if you simply applied Newton’s laws to a “particle” moving at the speed of light. This effect, though still very small, has been measured accurately.

Einstein’s theory has remarkable consequences when gravity is strong – wherever, in Newton’s theory, the orbital speeds would be a good fraction of the speed of light. Here the underlying concept that particles take the “straightest” path through curved space – which offers a deeper insight into gravity even when it is weak – becomes indispensable.

To understand the concept of “curvature,” it is helpful to think first of how we could check whether an ordinary two-dimensional surface was curved or flat. Suppose you drew two circles of latitude around the North Pole of the Earth, one slightly larger than the other, and measured their circumferences by crawling round them. If the Earth were flat,

the difference would be  $2\pi x$ , when  $x$  is the difference between the radii. However, on the Earth, where  $x$  is measured along a meridian of longitude, the difference would be less than  $2\pi x$ . Measurements of this kind can tell two-dimensional beings whether the surface they live on is curved.

The curvature of three-dimensional space around a point mass can be understood in the same way: the circumferences of circles



**Concentric circles drawn on a plane differ in circumference by an amount equal to  $2\pi$  times their difference in radius. If we drew concentric circles on a sphere and measured their radii along the surface of the sphere, the difference in circumference would be smaller than  $2\pi$  times their difference in radius. This discrepancy arises because the sphere is curved, whereas the plane is flat.**

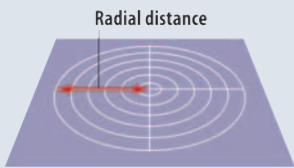


General Relativity and Black Holes

Whereas the gravitational deflection of light is a very small effect in the Solar System and in most astronomical contexts, light is severely bent in the strongly curved space close to the Schwarzschild radius. Indeed, light is even more strongly perturbed than around a neutron star. If our astronauts could hover just outside the Schwarzschild radius, they would need to aim their radio beam almost directly outward to keep it from being dragged back and eventually swallowed by the black hole. If they ventured within this radius, they would be unable to send any signals whatever to the outside world.

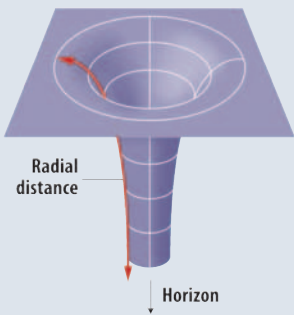
aren't  $2\pi r$ , where  $r$  is the radial distance. Schwarzschild's equations tell us what the curvature is – how the circumference differs from  $2\pi r$ , and how it depends on radius.

According to these equations, a bizarre thing happens to the circumference when the radius reaches  $3M$  kilometers, where  $M$  is the central mass in solar masses. Instead of continuing to decrease with decreasing radius, as we would expect,



**Close to a Schwarzschild black hole, space is severely stretched in the radial direction compared to the perpendicular directions. Because of the curvature of space, an object falling inward never seems to get**

the circumference levels off and becomes independent of radius! This means that, to a distant observer, an object can keep falling and falling toward the center of gravity and never appear to get any closer. In other words, its inward motion seems to "freeze out" when it reaches  $3M$  kilometers. The reader has probably noted that this radius is precisely the radius of the black hole's horizon; indeed, what we have just described is an interpretation of



**any closer to the center of the hole than the radius of the horizon. This three-dimensional effect can be represented in two dimensions by a so-called embedding diagram that resembles the mouth of a trumpet.**

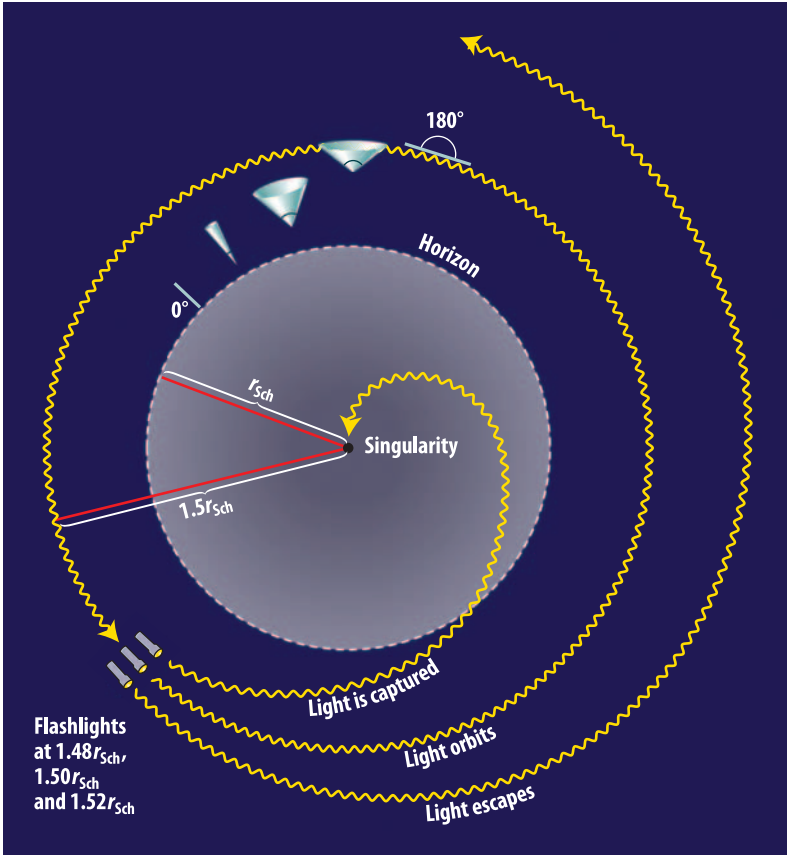
the horizon in terms of the curvature of space.

Where does the numerical value of the horizon's radius come from? Using Schwarzschild's equations, a physicist might write the "Schwarzschild radius" in the form  $2GM/c^2$ . Here,  $M$  is the central mass,  $c$  is the speed of light, and  $G$  is the "universal constant of gravitation" from Newton's theory.

Schwarzschild's equations imply that gravity, in effect, increases with decreasing distance more steeply than the inverse square law of Newton – it becomes more than 4 times stronger when the distance halves. A particle close to the central mass needs more energy than in Newtonian theory in order to stay in orbit. When an object is orbiting at less than 3 times the Schwarzschild radius, this effect is so drastic that a slight nudge to a body would send it spiraling toward the center. (In Newtonian theory, conservation of angular momentum would prevent this from happening.) Indeed, at 1.5 Schwarzschild radii, a body would have to move at the speed of light to stay in orbit.

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Trajectories of light rays become severely curved close to the horizon of a black hole. As the horizon is approached, a light beam must be aimed within an increasingly narrow cone in order to avoid being dragged into the hole. At 1.5 Schwarzschild radii a carefully aimed light beam can orbit the black hole indefinitely. At the horizon, the escape cone closes up – no light rays emitted from this radius can avoid being sucked into the black hole. Radiation from just outside the Schwarzschild radius would reach a distant observer with a large redshift, and a clock near the hole would appear to run slow.



Although the region enclosed within the horizon is shrouded from an external observer's view, the explorers would initially find nothing unusual as they passed through the critical radius. However, they would then have entered a region from which, however hard they fired their rockets, they could never escape. They would be inexorably drawn to a central point – the “singularity” – where tidal forces (the difference in the gravitational acceleration from one side of their spaceship to the other) would become infinite, and they and their ship would be crushed out of existence. As measured by their own clock, the trip from the horizon to the singularity would have taken just about as long as the time required for a light beam to travel the length of the Schwarzschild radius. But an external observer would not even see the astronauts cross the Schwarzschild radius: as they approached the horizon, their clock would appear to the distant observer to run slower and slower, and any signal that they sent would become more and more redshifted. Photons –