CLASSICAL MEASUREMENTS IN CURVED SPACE-TIMES

The theory of relativity describes the laws of physics in a given space-time. However, a physical theory must provide observational predictions expressed in terms of measurements, which are the outcome of practical experiments and observations.

Ideal for researchers with a mathematical background and a basic knowledge of relativity, this book will help in the understanding of the physics behind the mathematical formalism of the theory of relativity. It explores the informative power of the theory of relativity, and shows how it can be used in space physics, astrophysics, and cosmology. Readers are given the tools to pick out from the mathematical formalism the quantities which have physical meaning, which can therefore be the result of a measurement. The book considers the complications that arise through the interpretation of a measurement which is dependent on the observer who performs it. Specific examples of this are given to highlight the awkwardness of the problem.

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Classical Measurements in Curved Space-Times

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Preface

A physical measurement is meaningful only if one identifies in a non-ambiguous way who is the observer and what is being observed. The same observable can be the target of more than one observer so we need a suitable algorithm to compare their measurements. This is the task of the theory of measurement which we develop here in the framework of general relativity.

Before tackling the formal aspects of the theory, we shall define what we mean by observer and measurement and illustrate in more detail the concept which most affected, at the beginning of the twentieth century, our common way of thinking, namely the relativity of time.

We then continue on our task with a review of the entire mathematical machinery of the theory of relativity. Indeed, the richness and complexity of that machinery are essential to define a measurement consistently with the geometrical and physical environment of the system under consideration.

Most of the material contained in this book is spread throughout the literature and the topic is so vast that we had to consider only a minor part of it, concentrating on the general method rather than single applications. These have been extensively analyzed in Clifford Will's book (Will, 1981), which remains an essential milestone in the field of experimental gravity. Nevertheless we apologize for all the references that would have been pertinent but were overlooked.

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Notation

$\Re$: The real line.

$\Re^4$: The space of the quadruplets of real numbers.

$\{x^\alpha\}|_{\alpha=0,1,2,3}$: A quadruplet of local coordinates.

$\{e_\alpha\}$: A field of bases (frames) for the tangent space.

$\{\omega^\alpha\}$: A field of dual bases (dual frames) $\omega^\alpha(e_\beta) = \delta^\alpha{}_\beta$.

$\mathrm{g} = g_{\alpha\beta}\omega^\alpha \otimes \omega^\beta$: The metric tensor.

g^{-1}: Inverse metric.

g: Determinant of the metric.

$X^\#$: A tangent vector field (with contravariant components).

$X^\flat$: The 1-form (with covariant components) g-isomorphic to X.

$\rfloor$ (left contraction): Contraction of the rightmost contravariant index of the first tensor with the leftmost covariant index of the second tensor, that is $[S \rfloor T]^{\cdots}{}_{\cdots} = S^{\cdots\alpha}T_{\alpha\cdots}$.

$\llcorner$ (right contraction): Contraction of the rightmost covariant index of the first tensor with the leftmost contravariant index of the second tensor, that is $[S \llcorner T]^{\cdots}{}_{\cdots} = S_{\cdots\alpha}T^{\alpha\cdots}$.

$\overset{p}{\rfloor}$ (left p-contraction): Contraction of the rightmost p contravariant indices of the first tensor with the leftmost p covariant indices of the second tensor, i.e. $S \overset{p}{\rfloor} T \equiv S^{\alpha\ldots\beta_1\ldots\beta_p}T_{\beta_1\ldots\beta_p\ldots}$.

$\overset{p}{\llcorner}$ (right p-contraction): Contraction of the rightmost p covariant indices of the first tensor with the leftmost p contravariant indices of the second tensor, i.e. $S \overset{p}{\llcorner} T \equiv S^{\alpha\cdots}{}_{\beta_1\ldots\beta_p}T^{\beta_1\ldots\beta_p\cdots}$.

$\binom{r}{s}$-tensor: A tensor r-times contravariant and s-times covariant.

$[\alpha_1 \ldots \alpha_p]$: Antisymmetrization of the p indices.

$(\alpha_1 \ldots \alpha_p)$: Symmetrization of the p indices.

$[\mathrm{ALT}\, S]_{\alpha_1\ldots\alpha_p} = S_{[\alpha_1\ldots\alpha_p]}$.

$[\mathrm{SYM}\, S]_{\alpha_1\ldots\alpha_p} = S_{(\alpha_1\ldots\alpha_p)}$.

$e_\gamma(\cdot)$: γ-component of a frame derivative.

∇: Covariant derivative.

∇_{e_α}: α-component of the covariant derivative relative to the frame $\{e_\sigma\}$.

$\epsilon_{\alpha_1\ldots\alpha_4} = \epsilon_{[\alpha_1\ldots\alpha_4]}$: Levi-Civita alternating symbol.

$\eta_{\alpha_1\ldots\alpha_4} = g^{1/2}\epsilon_{\alpha_1\ldots\alpha_4}$; $\eta^{\alpha_1\ldots\alpha_4} = -g^{-1/2}\epsilon^{\alpha_1\ldots\alpha_4}$: The unit volume 4-form.

$\delta^{\alpha_1\ldots\alpha_4}{}_{\beta_1\ldots\beta_4} = \epsilon^{\alpha_1\ldots\alpha_4}\epsilon_{\beta_1\ldots\beta_4} = -\eta^{\alpha_1\ldots\alpha_4}\eta_{\beta_1\ldots\beta_4}$: Generalized Kronecker delta.

$[{}^*S]_{\alpha_{p+1}\ldots\alpha_4} = \frac{1}{p!}\, S_{\alpha_1\ldots\alpha_p}\, \eta^{\alpha_1\ldots\alpha_p}{}_{\alpha_{p+1}\ldots\alpha_4}$: Hodge dual of $S_{\alpha_1\ldots\alpha_p}$.

"$\cdot$": Scalar g-product, i.e. $u\cdot v = g(u,v) = g_{\alpha\beta}u^\alpha v^\beta$ for any pair of vectors $(u,\, v)$.

$\wedge$: The exterior or wedge product, i.e. $u\wedge v = u\otimes v - v\otimes u$ for any pair $(u,\, v)$ of vectors or 1-forms.

$\{e_{\hat\alpha}\}$: An orthonormal frame (tetrad).

$\frac{D}{ds}$: Absolute derivative along a curve γ with parameter s, i.e. $D/ds = \nabla_{\dot\gamma}$.

$a(u)$: Acceleration vector of the world line with tangent vector field u, i.e. $a(u) = \nabla_u u$.

$\frac{D_{(\mathrm{fw},u)}}{ds}$: The Fermi-Walker derivative along the curve with parameter s. For any vector field X: $\frac{D_{(\mathrm{fw},u)}\,X}{ds} = \frac{DX}{ds} \pm [a(u)(u\cdot X) - u(a(u)\cdot X)]$.

$\mathcal{C}_X$: The congruence of curves with tangent field X.

$\omega(X)$: The vorticity tensor of the congruence $\mathcal{C}_X$ (the same symbol also denotes the vorticity vector).

$\theta(X)$: The expansion tensor of the congruence $\mathcal{C}_X$.

$\Theta(X) = \mathrm{Tr}\,\theta(X)$: The trace of the expansion tensor of the congruence $\mathcal{C}_X$.

$\pounds_X$: Lie derivative along the congruence $\mathcal{C}_X$.

$C^\gamma{}_{\alpha\beta}$: Structure functions of a given frame.

$\omega^{\alpha_1\ldots\alpha_p} = p!\omega^{[\alpha_1}\otimes\cdots\otimes\omega^{\alpha_p]}$: The dual basis tensor of a space of p-forms.

$\delta T = {}^*d[{}^*T]$: Divergence of a p-form T.

$\Delta_{(\mathrm{dR})} = \delta d + d\delta$: de Rham operator.

LRS_u: Local rest space of u.

$P(u)$: u-spatial projector operator which generates LRS_u.

$T(u)$: u-temporal projector operator which generates the time axis of u.

$[P(u)S] \equiv S(u)$: Total u-spatial projection of a tensor S such that

$$[S(u)]^{\alpha_1 \cdots}{}_{\beta_1 \dots} = P(u)^{\alpha_1}{}_{\sigma_1} \cdots P(u)^{\rho_1}{}_{\beta_1} \cdots S^{\sigma_1 \cdots}{}_{\rho_1 \dots}.$$

$[\pounds(u)_X]$: u-spatially projected Lie derivative. For any tensor T it is

$$[\pounds(u)_X T]^{\alpha \cdots}{}_{\beta \dots} = P(u)^{\alpha}_{\sigma} \cdots P(u)^{\rho}{}_{\beta} \cdots [\pounds_X T]^{\sigma} \cdots{}_{\rho \dots}.$$

$\nabla(u)_{\rm lie} = \pounds(u)_u$: u-spatial Lie temporal derivative.

$\nabla(u) = P(u)\nabla$: u-spatially projected covariant derivative.

$P(u)\frac{D_{({\rm fw},X)}}{ds}$: u-spatially projected Fermi-Walker derivative along a curve with unit tangent vector X.

$d(u) = P(u)d$: u-spatially projected exterior derivative.

"$\cdot_u$": u-spatial inner product, i.e. $X \cdot_u Y = P(u)_{\alpha\beta} X^\alpha Y^\beta$.

"$\times_u$" : u-spatial cross product, i.e. $[X \times_u Y]^\alpha = \eta(u)^\alpha{}_{\rho\sigma} X^\rho Y^\sigma$.

$\eta(u)^\alpha{}_{\rho\sigma} = u_\beta \eta^{\beta\alpha}{}_{\rho\sigma}$: u-spatial 4-volume.

${\rm grad}_u = \nabla(u)$: u-spatial gradient.

${\rm curl}_u = \nabla(u)\times_u$: u-spatial curl.

${\rm div}_u = \nabla(u)\cdot_u$: u-spatial divergence.

${\rm Scurl}_u$: Symmetrized ${\rm curl}_u$, i.e. $[{\rm Scurl}_u\, A]^{\alpha\beta} = \eta(u)^{\gamma\delta(\alpha} \nabla(u)_\gamma A^{\beta)}{}_\delta$.

$C_{({\rm fw})ab}$: Fermi-Walker rotation coefficients, i.e. $C_{({\rm fw})ab} = e_b \cdot \nabla_u e_a$.

$C_{({\rm lie})}{}^b{}_a$: Lie rotation coefficients, i.e. $C_{({\rm lie})}{}^b{}_a = \omega^b(\pounds(u)_u e_a)$.

$\nabla(u)_{({\rm fw})} = P(u)\nabla_u$: u-spatial Fermi-Walker temporal derivative.

$\nabla(u)_{({\rm tem})}. \equiv \nabla(u)_{({\rm fw})}$ or $\nabla(u)_{({\rm lie})}$

$\nu(U,u)$: Relative spatial velocity of U with respect to u.

$\nu(u,U)$: Relative spatial velocity of u with respect to U.

$\gamma(U,u) = \gamma(u,U) = \gamma$: Lorentz factor of the two observers u and U.

$\hat\nu(u,U)$: Unitary relative velocity vector of u with respect to U.

ζ: Angular velocity.

$\omega(k,u)$: Frequency of the light ray k with respect to the observer u.

$||\nu(U,u)|| = ||\nu(u,U)|| = \nu$: Magnitude of the relative velocity of the two observers u and U.

$B(U, u)$: Relative boost from u to U.

$P(U, u) = P(U)P(u)$: Mixed projector operator from LRS_u into LRS_U.

$B_{(\mathrm{lrs})}(U, u) = P(U)B(U, u)P(u)$: Boost from LRS_u into LRS_U.

$B_{(\mathrm{lrs})}(U, u)^{-1} = B_{(\mathrm{lrs})}(u, U)$: Inverse boost from LRS_U to LRS_u.

$B_{(\mathrm{lrs})_u}(U, u) = P(U, u)^{-1} \llcorner B_{(\mathrm{lrs})}(U, u)$.

$\frac{D_{(\mathrm{lie}, U)} X}{d\tau_U} = [U,\ X]$: Lie derivative of X along $\mathcal{C}_U$.

$\tau_{(U,\,u)}$: Relative standard time parameter, i.e. $d\tau_{(U,\,u)} = \gamma(U,\ u) d\tau_U$.

$\ell_{(U,\,u)}$: Relative standard length parameter, i.e. $d\ell_{(U,\,u)} = \gamma(U,\ u)||\nu(U,\ u)||d\tau_U$.

$\frac{D_{(\mathrm{fw}, U, u)}}{d\tau_{(U,\,u)}} = P(u)\frac{D}{d\tau_{(U,\,u)}}$: Projected absolute covariant derivative along U.

$a_{(\mathrm{fw}, U, u)} = P(u)\frac{D\nu(U,u)}{d\tau_{(U,u)}}$: Relative acceleration of U with respect to u.

$(\nabla X)_{\alpha\beta} \equiv \nabla_\beta X_\alpha$.

Physical dimensions

We are using geometrized units with $G = 1 = c$, G and c being Newton's gravitational constant and the speed of light in vacuum, respectively. Symbols are as they appear in the text; the reader is advised that more than one symbol may be used for the same item and conversely the same symbol may refer to different items. Reference is made to the observers $(U,\ u)$.

Time	t	$\rightarrow$	$[L]^1$
Space	r, x, y, z	$\rightarrow$	$[L]^1$
Mass	$\mathcal{M}, m, \mu_0$	$\rightarrow$	$[L]^1$
Angular velocity	ζ	$\rightarrow$	$[L]^{-1}$
Energy	E	$\rightarrow$	$[L]^1$
Specific energy (in units of mc^2)	E, γ	$\rightarrow$	$[L]^0$
Specific angular momentum (in units of mc)	$L, \lambda, \Lambda, \ell$	$\rightarrow$	$[L]^1$
of a rotating source (Kerr)	a	$\rightarrow$	$[L]^1$
Spin	S	$\rightarrow$	$[L]^2$
Specific spin	S/m	$\rightarrow$	$[L]^1$
4-velocity	U, u	$\rightarrow$	$[L]^0$
Relative velocity	$\nu(U, u)$	$\rightarrow$	$[L]^0$
Force	F	$\rightarrow$	$[L]^0$
Acceleration	$a(U)$	$\rightarrow$	$[L]^{-1}$
Expansion	$\Theta(U)$	$\rightarrow$	$[L]^{-1}$

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Vorticity	$(\omega(U)_{\alpha\beta}\,\omega(U)^{\alpha\beta})^{1/2}$	$\rightarrow$	$[L]^{-1}$
Electric charge	Q, e	$\rightarrow$	$[L]^{1}$
Space-time curvature	$(R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta})^{1/2}$	$\rightarrow$	$[L]^{-2}$
Electric field	$E(U)$	$\rightarrow$	$[L]^{-1}$
Magnetic field	$B(U)$	$\rightarrow$	$[L]^{-1}$
Frequency	$\omega(k, u)$	$\rightarrow$	$[L]^{-1}$
Gravitational wave amplitude	$h_{+,\times}$	$\rightarrow$	$[L]^{0}$
Strain	$S(U)$	$\rightarrow$	$[L]^{-2}$.

Conversion factors

We list here conversion factors from conventional CGS to geometrized units. For convenience we denote the quantities in CGS units with a tilde ($\sim$).

Name	CGS units	Geometrized units
Mass	$\tilde{M}$	$\mathcal{M} = \frac{G\tilde{M}}{c^2}$
Electric charge	$\tilde{Q}$	$Q = \tilde{Q}\sqrt{\frac{G}{4\pi\epsilon_0 c^4}}$
Velocity	$\tilde{v}$	$\nu = \frac{\tilde{v}}{c}$
Acceleration	$\tilde{a}$	$a = \frac{\tilde{a}}{c^2}$
Force	$\tilde{F}$	$F = \frac{G\tilde{F}}{c^4}$
Electric field	$\tilde{E}$	$E = \tilde{E}\sqrt{\frac{4\pi\epsilon_0 G}{c^4}}$
Magnetic field	$\tilde{H}$	$H = \tilde{H}\sqrt{\frac{4\pi\epsilon_0 G}{c^2}}$
Energy	$\tilde{\mathcal{E}}$	$\mathcal{E} = \frac{G\tilde{\mathcal{E}}}{c^4}$
Specific energy	$\frac{\tilde{\mathcal{E}}}{\tilde{M}c^2}$	$\frac{\mathcal{E}}{\mathcal{M}} \equiv \gamma = \frac{\tilde{\mathcal{E}}}{\tilde{M}c^2}$
Angular momentum	$\tilde{L}$	$L = \frac{G\tilde{L}}{c^3}$
Angular momentum in units of $\tilde{M}c$	$\frac{\tilde{L}}{\tilde{M}c}$	$\lambda = \frac{L}{\mathcal{M}} = \frac{\tilde{L}}{\tilde{M}c}$