Entropy in Dynamical Systems

This comprehensive text on entropy covers three major types of dynamics: measure-preserving transformations; continuous maps on compact spaces; and operators on function spaces.

Part I contains proofs of the Shannon–McMillan–Breiman Theorem, the Ornstein–Weiss Return Time Theorem, the Krieger Generator Theorem, the Sinai and Ornstein Theorems, and among the newest developments, the Ergodic Law of Series. In Part II, after an expanded exposition of classical topological entropy, the book addresses Symbolic Extension Entropy. It offers deep insight into the theory of entropy structure and explains the role of zero-dimensional dynamics as a bridge between measurable and topological dynamics. Part III explains how both measure-theoretic and topological entropy can be extended to operators on relevant function spaces.

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CAMBRIDGE
UNIVERSITY PRESS

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University Printing House, Cambridge CB2 8BS, United Kingdom

Published in the United States of America by Cambridge University Press, New York

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www.cambridge.org
Information on this title: www.cambridge.org/9780521888851

First published 2011 (Twice)
Second Edition 2012
Reprinted 2013

A catalogue record for this publication is available from the British Library

Library of Congress Cataloguing in Publication data

Downarowicz, Tomasz, 1956–
Entropy in Dynamical Systems / Tomasz Downarowicz.
p. cm. – (New Mathematical Monographs ; 18)
Includes bibliographical references and index.
ISBN 978-0-521-88885-1 (Hardback)
1. Topological entropy–Textbooks. 2. Topological dynamics–Textbooks. I. Title.
QA611.5.D685 2011
515'.39–dc22
2010050336

ISBN 978-0-521-88885-1 Hardback

To my Parents

Cambridge University Press
978-0-521-88885-1 - Entropy in Dynamical Systems
Tomasz Downarowicz
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Preface

This book is designed as a comprehensive lecture on entropy in three major types of dynamics: measure-theoretic, topological and operator. In each case the study is restricted to the most classical case of the action of iterates of a single transformation (or operator) on either a standard probability space or on a compact metric space. We do not venture into studying actions of more general groups, dynamical systems on noncompact spaces or equipped with infinite measures. On the other hand, we do not restrict the generality by adding more structure to our spaces. The most structured systems addressed here in detail are smooth transformations of the compact interval. The primary intention is to create a self-contained course, from the basics through more advanced material to the newest developments. Very few theorems are quoted without a proof, mainly in the chapters or sections marked with an asterisk. These are treated as "nonmandatory" for the understanding of the rest of the book, and can be skipped if the reader chooses. Our facts are stated as generally as possible within the assumed scope, and wherever possible our proofs of classical theorems are different from those found in the most popular textbooks. Several chapters contain very recent results for which this is a textbook debut.

We assume familiarity of the reader with basics of ergodic theory, measure theory, topology and functional analysis. Nevertheless, the most useful facts are recalled either in the main text or in the appendix.

Some elementary statements and minor passages are left without a proof, as an exercise for the reader. Such statements are collected at the end of each chapter, together with other exercises of independent interest. It is planned that solutions to selected exercises will be made available shortly after the book has occurred in print, at the publisher's website www.cambridge.org/9780521888851.

Acknowledgments

First of all, I wish to express my gratitude to those who helped me with the mathematical issues, especially to Mike Boyle, Dan Rudolph, Jean-Paul Thouvenot and Benjy Weiss. I thank all those who read the preliminary version and helped me fix the mistakes. Large portions were proofread by Bartosz Frej, Jacek Serafin and David Burguet. I am grateful to the authorities of my Institute and Faculty for their understanding and a reduction on other university duties allowing me to better focus on writing the book. My warmest thanks are directed to Sylwia for her love and priceless help in organizing my work and everyday life during this busy time.

The research of the author was partially supported by the Polish Ministry of Education Grant Number N N201 394537.