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Mathematical Tools for One-Dimensional Dynamics

Originating with the pioneering works of P. Fatou and G. Julia, the subject of complex dynamics has seen great advances in recent years. Complex dynamical systems often exhibit rich, chaotic behavior, which yields attractive computer generated pictures, for example the Mandelbrot and Julia sets, which have done much to renew interest in the subject.

In this self-contained book, the major mathematical tools necessary for the study of complex dynamics at an advanced level are discussed. Complete proofs of some of these tools are presented; some, such as the Bers–Royden theorem on holomorphic motions, appear for the first time in book format. Riemann surfaces and Teichmüller theory are considered in an appendix. Detailing the latest research, the book will appeal to graduate students and researchers working in dynamical systems and related fields. Carefully chosen exercises aid understanding and provide a glimpse of further developments in real and complex one-dimensional dynamics.

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Mathematical Tools for One-Dimensional Dynamics

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Preface

Our main goal in this book is to introduce the reader to some of the most useful tools of modern one-dimensional dynamics. We do not aim at being comprehensive but prefer instead to focus our attention on certain key tools. We believe that the topics covered here are representative of the depth and beauty of the ideas in the subject. For each tool presented in the book, we have selected at least one non-trivial dynamical application to go with it.

Almost all the topics discussed in the text have their source in complex function theory and the related areas of hyperbolic geometry, quasiconformal mappings and Teichmüller theory. This is true even of certain tools, such as the distortion of cross-ratios, that are applied to problems in *real* one-dimensional dynamics. The main tools include three deep theorems: the *uniformization theorem* (for domains in the Riemann sphere), the *measurable Riemann mapping theorem* and the *Bers–Royden theorem* on holomorphic motions. These are presented, with complete proofs, in Chapters 3, 4 and 5 respectively.

The present book originated in a set of notes for a short course we taught at the 23rd Brazilian Mathematics Colloquium (IMPA, 2001). We have benefited from useful criticism of the original notes from friends and colleagues, especially André de Carvalho, who read through them and found several inaccuracies. We are also grateful to two anonymous referees for their perspicacious remarks and suggestions.

The drawings of Julia sets and the Mandelbrot set found in this book were made with the help of computer programs written by C. Mullen (available through his homepage, at `www.math.harvard.edu/~ctm`). We wish to thank also Dayse H. Pastore for her help with the figures for the original colloquium notes, some of which appear in the present book.

Finally, it is quite an honor to see our book published by Cambridge University Press, and in such a prestigious series. We are grateful to David Tranah and Peter Thompson for this opportunity and for their patient and highly professional support.

Edson de Faria and Welington de Melo
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