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978-0-521-88851-6 - The Large Sieve and its Applications: Arithmetic Geometry, Random Walks and Discrete Groups

E. Kowalski

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