INTRODUCTION TO NUMERICAL GEODYNAMIC MODELLING

Until now, numerical modelling of geodynamic processes has been the domain of highly trained mathematicians with long experience of numerical and computational techniques. Now, for the first time, students and new researchers in the Earth Sciences can learn the basic theory and applications from a single, accessible reference text.

Assuming only minimal prerequisite mathematical training (simple linear algebra and derivatives) the author provides a solid grounding in the basic mathematical theory and techniques, including continuum mechanics and partial differential equations, before introducing key numerical and modelling methods. Eight well-documented and state-of-the-art visco-elasto-plastic, 2D models are then presented, which allow robust modelling of key dynamic processes such as subduction, lithospheric extension, collision, slab break-off, intrusion emplacement, mantle convection and planetary core formation.

Incorporating 47 practical exercises and 67 MATLAB examples (for which codes are available online at www.cambridge.org/gerya) this textbook provides a user-friendly introduction for graduate courses or self-study, and encourages readers to experiment with geodynamic models first hand.

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