Flow, Deformation and Fracture

Over 40 years of teaching experience are distilled into this text. The guiding principle is the wide use of the concept of intermediate asymptotics, which enables the natural introduction of the modeling of real bodies by continua.

Beginning with a detailed explanation of the continuum approximation for the mathematical modeling of the motion and equilibrium of real bodies, the author continues with a general survey of the necessary methods and tools for analyzing models. Next, specific idealized approximations are presented, including ideal incompressible fluids, elastic bodies and Newtonian viscous fluids. The author not only presents general concepts, but also devotes chapters to examining significant problems, including turbulence, wave propagation, defects and cracks, fatigue and fracture. Each of these applications reveals essential information about the particular approximation.

The author's tried and tested approach reveals insights that will be valued by every teacher and student of mechanics.

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Flow, Deformation and Fracture

Lectures on Fluid Mechanics and the Mechanics of Deformable Solids for Mathematicians and Physicists

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University Printing House, Cambridge CB2 8BS, United Kingdom

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www.cambridge.org
Information on this title: www.cambridge.org/9780521887526

First published 2014

Printed in the United Kingdom by CPI Group Ltd, Croydon CR0 4YY

A catalogue record for this publication is available from the British Library

Library of Congress Cataloguing in Publication data
Barenblatt, G. I.
Flow, deformation and fracture : lectures on fluid mechanics and the mechanics of deformable solids for mathematicians and physicists / Grigory Isaakovich Barenblatt, for. mem. RS, emeritus professor, University of California, Berkeley, emeritus G.I. Taylor Professor of Fluid Mechanics, University of Cambridge, principal scientist, Institute of Oceanology, Russian Academy of Sciences.
pages cm. – (Cambridge texts in applied mathematics)
Includes bibliographical references and index.
ISBN 978-0-521-88752-6 (hardback) – ISBN 978-0-521-71538-6 (pbk.) 1. Fluid mechanics – Textbooks. 2. Deformations (Mechanics) – Textbooks. 3. Fracture mechanics – Textbooks. I. Title.
QA901.B37 2014
532′.05 – dc23 2013048955

ISBN 978-0-521-88752-6 Hardback
ISBN 978-0-521-71538-6 Paperback

To the glowing memory of my grandfather,

Professor at Moscow State University,

Veniamin Fedorovich Kagan

this work is dedicated

with eternal love and gratitude

Contents

Foreword

In his preface to this book, Professor G. I. Barenblatt recounts the saga of the course of mechanics of continua on which the book is based. This saga originated at the Moscow State University under the aegis of the renowned Rector I. G. Petrovsky and moved with the author first to the Moscow Institute for Physics and Technology, then to Cambridge University in England, then to Stanford University, until it reached its final home as a much loved and appreciated course at the mathematics department of the University of California, Berkeley. Those not fortunate enough to have been able to attend the course now have the opportunity to see what has made it so special.

The present book is a masterful exposition of fluid and solid mechanics, informed by the ideas of scaling and intermediate asymptotics, a methodology and point of view of which Professor Barenblatt is one of the originators. Most physical theories are intermediate, in the sense that they describe the behavior of physical systems on spatial and temporal scales intermediate between much smaller scales and much larger scales; for example, the Navier–Stokes equations describe fluid motion on spatial scales larger than molecular scales but not so large that relativity must be taken into account and on time scales larger than the time scale of molecular collisions but not so large that the vessel that contains the fluid collapses through aging. An awareness of the scales that are relevant to each problem must guide the formulation of mathematical models as well as the asymptotics that lead to their solution. Accordingly, the book makes explicit the intermediate asymptotic nature of many of the well-known arguments in mechanics, leading to a clear understanding of their domains of validity and their limitations. Along the way, the assumptions that underlie the various models are spelled out in detail and expressed in terms of the appropriate dimensionless numbers. Dimensional and scaling considerations are introduced early, allowing the reader an easy way to understand the consequences of the various assumptions without the heavy mathematical machinery that can impede understanding if it is introduced prematurely.

These unique features make it possible for this book to present, without long preliminaries, not only the basic features of the mechanics of continua but also more sophisticated topics in turbulence and fracture, fields to which Professor Barenblatt has contributed so much but where present understanding is still incomplete. In particular, the reader will surely appreciate the illuminating presentations of crack propagation and of the scaling of turbulent shear flow. I was privileged to work on this last topic with Professors Barenblatt and Prostokishin, and I feel very strongly that our straightforward scaling arguments satisfy all logical requirements and fit all the data ("all" means literally "all") better than the older alternatives. The presentation here is detailed enough for the reader to form his or her own opinion.

Finally, I would like to mention that the historical asides that Professor Barenblatt provides throughout are enjoyable and illuminating. This is indeed a remarkable book.

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University of California

Cambridge University Press
978-0-521-88752-6 - Flow, Deformation and Fracture: Lectures on Fluid Mechanics and the Mechanics of Deformable Solids for Mathematicians and Physicists
Grigory Isaakovich Barenblatt
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Preface

(1) Mechanics, the science of the motion and equilibrium of real bodies, is the oldest natural science created by humankind. Unlike other scientific disciplines, mechanics had no predecessors. Prehistoric human beings started to take their first steps in mechanics – apparently even before starting to speak – when they invented and improved their first primitive tools.

Later, when humankind took its first steps in mathematics, mechanics was the first field of application. The value of mechanics for mathematics was clearly emphasized by Leonardo da Vinci: “Mechanics is the paradise for mathematical sciences because through it one comes to the fruits of mathematics.”

Nowadays mechanics is an organic part of applied mathematics – the art of designing mathematical models of phenomena in nature, society and engineering.

The development of mechanics, and the recognition of its value, has not gone smoothly over the centuries. An analogy of mechanics with the phoenix comes to mind. This legendary bird has appeared with practically identical magical features in the ancient legends of many cultures: Egyptian, Chinese, Hebrew, Greek, Roman, native North American, Russian and others. The names were different: the name “phoenix” was coined by the Assyrians; Russians called it “zhar-ptitsa” (fire-bird). According to the legend, unlike all other living beings the phoenix had no parents, and death could never touch it. However, from time to time, when it was weakened, the phoenix would carefully prepare a fire from aromatic herbs collected from throughout the world and burn itself. Everything superfluous is burnt in the fire and a new beautiful creative life opens to the phoenix.

So, what is the analogy? Mechanics now is living through a critical period. The community at large, particularly the scientific community, often considers mechanics to be a subject of secondary value. As a practical result of this attitude, bright young people nowadays are not interested in choosing mechanics as their profession. Inevitably, and rather quickly, a decline in the level of students leads to a decline in the level of professors. One after another mechanics departments in universities and technological institutes close; mechanics disappears from the curricula of physics and mathematics.

I am sure that a phoenix-type rebirth of mechanics is unavoidable. The reason for my confidence is the existence (not always generally recognized by society, nor its political leaders) of fundamental problems of vital importance for humankind that cannot be solved without the leading participation of mechanics and applied mathematics as a whole. To mention a few of these: the suppression of tropical hurricanes; the prediction of earthquakes three to four hours before the event; the creation of a new branch of engineering based on nano-technology; the creation of a new standard for the development of the deposits of the Earth's non-renewable resources, in particular oil, gas and coal, replacing the existing predatory exploitation; etc. These problems should be the subject of national and multinational programs. The participation of mechanics, renewed when necessary, in solving such problems is unavoidable: the strength and long-lastingness of mechanics come from its ability for continual renewal and, once renewed, for tackling potential problems of primary importance for leading nations and for humankind as a whole. The surge of interest in mechanics during World War II and its aftermath demonstrated this clearly.

(2) Ivan Georgievich Petrovsky, a great mathematician whose rectorship guided an epoch for Moscow State University, was a man of unsurpassable vision in science and education. His role as a leading mathematician is illustrated by a remarkable event: Sir Michael Atiyah, R. Bott and L. Gårding had published an article (Atiyah, Bott and Gårding, 1970) with a dedication to I. G. Petrovsky in Russian, clarifying and generalizing – as the authors said – Petrovsky's theory of lacunas for hyperbolic differential operators. His textbooks on ordinary differential equations (Petrovsky, 1966), partial differential equations (Petrovsky, 1967) and integral equations (Petrovsky, 1967) have seen wide use throughout the world. Petrovsky died in the building of the Central Committee of the Communist Party (he himself was not a member of the Party), fighting for his principles regarding the selection, teaching and education of students.

Petrovsky had a clear view of the role of mechanics in scientific education. It was he who offered the late V. I. Arnold, at that time a brilliant young professor of mathematics who became later one of the world's leading mathematicians, a chance to deliver the course of classical rational mechanics for student mathematicians. The mechanics community at Moscow State University considered this offer to be a risky experiment, but the result exceeded all expectations: Arnold's book (Arnold, 1978) *Mathematical Methods of Classical Mechanics* became a gem of scientific literature.[1]

[1] The title of the book was also a result of compromise.

Later, Petrovsky, as Chairman of the Department of Differential Equations, gave me the chance to teach the course on the mechanics of continua. The class was excellent: it contained very strong students, now disseminated over the entire planet. Ivan Georgievich offered me remarkable conditions: a one-year course, two exams, complete freedom in selecting the curriculum. However, he did impose one strict condition: "I cannot force the students to attend your lectures, so at each lecture you should tell them something special that they cannot find in the textbooks".

After the end of the Petrovsky era at Moscow State University I continued, through the initiative of my former student V. B. Librovich (at that time the leading person in the Academy's Institute for Problems in Mechanics), to deliver this course under the same conditions at the Moscow Institute for Physics and Technology (MIPT). This institute was a remarkable school, founded after World War II by an enthusiastic group of first-class physicists, applied mathematicians and engineers, headed by S. A. Christianovich. They were inspired by the example of the Ecole Polytechnique, and the level of the entering students was very high.

When I moved to the West, I delivered appropriate parts of this course in Cambridge, UK, the University of Illinois, Urbana–Champaign, Stanford University, and finally at the University of California, Berkeley. Naturally, each time the content of the course was suitably modified, but the general style that had been blessed by I. G. Petrovsky remained the same.

(3) Now I want to mention some specific features of the present book. Contrary to common practice, I do not devote a substantial initial part of the book to the presentation of tensor calculus. I prefer to introduce tensors and discuss their properties at the spot where they naturally appear.

At the very beginning of this book I introduce the concept of *intermediate asymptotics* and use it widely throughout. It is worthwhile to present here the definition of this concept, formally introduced by Ya. B. Zeldovich and myself, but *de facto* used long before that. Let us assume that in a problem under consideration there are two parameters X_1 and X_2 having the dimension of an independent variable x and that X_1 is much less than X_2, so that there exists an interval of values of x where $X_1 \ll x$ and at the same time $x \ll X_2$. The asymptotic representation, or simply asymptotics, in this interval is called the *intermediate asymptotics*.

It is emphasized that the fundamental concept of a *continuous medium* introduced after that, and widely discussed and illustrated by many examples, is one of intermediate asymptotics.

Also, at the very beginning, the concept of an *observer* is formally introduced, as well as the *invariance principle*: all physically significant relations can be written in a form valid for all observers.

(4) Next comes a detailed presentation of *dimensional analysis and physical similitude*, as a consequence of the invariance principle: physical relations can be written in a form valid for *all observers, having units of measurement of the same physical nature but of different magnitudes.* Dimensional analysis is widely used throughout the whole book, and it greatly simplifies the presentation. It is difficult for me to understand the custom of presenting these concepts at the end of mechanics of continua courses, where they can be used for nothing.

After introducing the fundamental concept of a continuous medium which, I emphasize, is based entirely on the intermediate asymptotic approach, multiple illustrations are presented justifying this concept on various scales from the atomic to the cosmic. After that the basic equations of mass and momentum conservation for continuous media are presented. Here the concept of the mass flux vector and the stress tensor appear naturally, following from the invariance principle.

(5) To obtain a closed system of basic equations for mathematical models, the continuous medium should be supplied with some physical features. A general approach using *finite* constitutive equations is outlined. It is emphasized that in fact this approach is of restricted value; it is justified only when the microstructure of the material remains intact in the process of the motion. Otherwise, the finite constitutive equations should be replaced by equations for the kinetics of microstructural transformation. This comment should be taken into account, in particular, when one is analyzing the rheology of complex fluids, principally polymeric solutions and melts. In particular, the universality of the constants entering constitutive equations such as the Oldroyd B model should be verified for selected materials in a given class of motions; otherwise, predictions based on the model could be incorrect.

(6) After these preliminary steps, basic models or idealizations are presented. It is emphasized that all these models are *approximations*, valid for a restricted class of materials in a restricted range of motions and loading situations.

After this we follow with the classic models (approximations): the model of an ideal incompressible fluid, the model of an ideal elastic solid, the model of a Newtonian viscous fluid, and the model of an ideal gas. The conditions of applicability of each model are discussed in detail. In their due place the conservation laws of energy and angular momentum are presented and used in constructing the models. These conservation laws appear exactly where they are needed and can be used.

(7) The presentation of each model follows the same general scheme: general relations are derived, with specially selected "accompanying problems", presented in detail with a discussion of why they can be used. So, for the model of an ideal incompressible fluid, the accompanying problems are the Lavrentiev problem of a

directed explosion, and the problem of the lift force on a slender, weakly inclined wing. It is demonstrated, in particular, that the model of an ideal incompressible fluid needs to be modified in a natural way in the formulation of the lift force problem, which is substantially nonlinear in spite of the linearity of the basic equations. Here the concept of intermediate asymptotics is crucially important. The problem of brittle and quasi-brittle fracture accompanies the presentation of the ideal elastic solid approximation. It is demonstrated that the model should also be thus modified for the formulation of this substantially nonlinear problem, in spite of the linearity of the basic equations. (The situation is similar to the lift force problem.) It is demonstrated that the fracture of a structure is not a local event but a global one. From a mathematical viewpoint, fracture is the loss of existence of the solution to an explicitly formulated nonlinear free boundary problem for the linear equations of the equilibrium of an elastic body.

The problem of the boundary layer and drag of a slender weakly inclined wing accompanies the Newtonian viscous fluid approximation. The Prandtl–Blasius intermediate asymptotic solution to the problem of the boundary layer on a flat plate is presented in detail; the underwater reefs to be found in this problem are specially emphasized and discussed.

A more detailed analysis and classification of scaling laws is needed before we can proceed further. This is presented in detail in Chapter 9. The concepts of complete and incomplete similarity are introduced and discussed. They are widely used in the subsequent chapters, in which we discuss the ideal gas approximation and turbulence. The accompanying problem of impulsive loading is presented in Chapter 10 concerning the ideal gas approximation.

The final two chapters concern turbulence. I have found that the only two accompanying problems preserving the same style as in previous chapters that can be presented are those of turbulent shear flows at very large Reynolds numbers and of the local structure of turbulent flows at very large Reynolds numbers (the Kolmogorov–Obukhov theory). The discussion of turbulent shear flows at very large Reynolds numbers is based on the works of A. J. Chorin, V. M. Prostokishin and the present author. Some colleagues consider our model to be controversial. That is their business; the formulae and the comparison with experimental data speak for themselves.

So, this is the content of the present book as it was delivered in my courses of lectures.

Now it is time for acknowledgments. I want to remember with deep gratitude and admiration the late Ivan Georgievich Petrovsky who generated this course at Moscow State University. The support and care of the late Vadim Bronislavovich Librovich allowed me to continue this course at the Moscow Institute for Physics

and Technology. I remember with gratitude Sergey Sergeevich Voyt who extended my audience at MIPT to the students affiliated with the Institute of Oceanology. I was honored by election to the G. I. Taylor Chair in Fluid Mechanics at the University of Cambridge: my cordial gratitude goes to G. K. Batchelor, D. G. Crighton, H. K. Moffatt and Sir James Lighthill. I spent a term at the University of Illinois, Urbana–Champaign: my deep thanks go to H. Aref and N. D. Goldenfeld. Later I was elected S. P. Timoshenko Visiting Professor of Applied Mechanics at Stanford University: my deep gratitude goes to M. D. Van Dyke, J. B. Keller and T. J. R. Hughes.

Before my period as Timoshenko Professor, I came for a short one-month Miller Visiting Professorship at Berkeley, by invitation of Professor Alexandre Chorin. We met for the first time on 16 February 1996; we both celebrate this date. In our first discussion we understood that our interests in turbulence studies and our general views on turbulence and science in general, although developed independently, practically coincide. We began to work together. Our first paper was submitted to the *Proceedings of the US National Academy of Sciences* three weeks later. Twenty papers followed this one and now, though working in different fields, we continue to systematically exchange thoughts. My gratitude to Alexandre is immeasurable: together with A. N. Kolmogorov and Ya. B. Zeldovich he became a benchmark in my scientific life. In particular I appreciate his many-fold advice concerning this book. Alexandre's efforts to obtain for me an honorary position at the University of California, Berkeley were strongly supported by Professor Calvin Moore, former Vice-President of the University of California, and at that time Chairman of the Department of Mathematics. Though himself a pure mathematician, Calvin Moore always strongly supported applied mathematics. I appreciate his friendship, kind attention and help rendered to me throughout my time at Berkeley.

I express my gratitude to Professor James Sethian, head of the Department of Mathematics at the Lawrence Berkeley National Laboratory. We work in different fields of mathematics, but his friendly attention to my work has been a permanent stimulus for me.

My cordial thanks go to Mrs Valerie Heartlie, Administrative Assistant of the department. She did her best to make this department a pleasant place for my work and everyday life.

I want to thank two people who played an essential role in my work on this book. These are Professor Valery Mikhailovich Prostokishin, who attended my lectures and gave me most valuable advice, and Dr David Tranah, Publishing Director, Mathematical Sciences and Information Technology, Cambridge University Press, who initiated the book, escorted it from its first version, and influenced its structure and style.

The help of Mrs Jean McKenzie, the Head Librarian of Berkeley Engineering Library, is highly appreciated.

I grew up in the family of my maternal grandfather, Veniamin Fedorovich Kagan (in the western mathematical literature he is known as Benjamin Kagan), after my mother, a physician–virologist, perished preparing the first vaccine for Japanese encephalitis. My grandfather was an outstanding mathematician, working in the foundations of geometry and non-Euclidean geometry; at Moscow State University he created an influential school of tensor differential geometry. He was a person of unbending principles in science and life and achieved an extraordinary moral authority, which allowed him to survive the stormy 1920s and 1930s of the Soviet era, even being accused of "sabotage in science" by the leading journal of the Communist party (*Bolshevik*) and thereafter immediately arrested.

The dedication of this book to his glowing memory is but a weak expression of my love and eternal gratitude to him.