

Contents

<i>List of figures</i>	<i>page</i> ix
<i>List of symbols</i>	x
Introduction	1
Historical Introduction	4
PART I: KAZHDAN'S PROPERTY (T)	25
1 Definitions, first consequences, and basic examples	27
1.1 First definition of Property (T)	27
1.2 Property (T) in terms of Fell's topology	32
1.3 Compact generation and other consequences	36
1.4 Property (T) for $SL_n(\mathbf{K})$, $n \geq 3$	40
1.5 Property (T) for $Sp_{2n}(\mathbf{K})$, $n \geq 2$	50
1.6 Property (T) for higher rank algebraic groups	58
1.7 Hereditary properties	60
1.8 Exercises	67
2 Property (FH)	73
2.1 Affine isometric actions and Property (FH)	74
2.2 1-cohomology	75
2.3 Actions on trees	80
2.4 Consequences of Property (FH)	85
2.5 Hereditary properties	88
2.6 Actions on real hyperbolic spaces	93
2.7 Actions on boundaries of rank 1 symmetric spaces	100
2.8 Wreath products	104
2.9 Actions on the circle	107
2.10 Functions conditionally of negative type	119

2.11	A consequence of Schoenberg's Theorem	122
2.12	The Delorme–Guichardet Theorem	127
2.13	Concordance	132
2.14	Exercises	133
3	Reduced cohomology	136
3.1	Affine isometric actions almost having fixed points	137
3.2	A theorem by Y. Shalom	140
3.3	Property (T) for $Sp(n, 1)$	151
3.4	The question of finite presentability	171
3.5	Other consequences of Shalom's Theorem	175
3.6	Property (T) is not geometric	179
3.7	Exercises	182
4	Bounded generation	184
4.1	Bounded generation of $SL_n(\mathbf{Z})$ for $n \geq 3$	184
4.2	A Kazhdan constant for $SL_n(\mathbf{Z})$	193
4.3	Property (T) for $SL_n(\mathbf{R})$	201
4.4	Exercises	213
5	A spectral criterion for Property (T)	216
5.1	Stationary measures for random walks	217
5.2	Laplace and Markov operators	218
5.3	Random walks on finite sets	222
5.4	G -equivariant random walks on quasi-transitive free sets	224
5.5	A local spectral criterion	236
5.6	Zuk's criterion	241
5.7	Groups acting on \tilde{A}_2 -buildings	245
5.8	Exercises	250
6	Some applications of Property (T)	253
6.1	Expander graphs	253
6.2	Norm of convolution operators	262
6.3	Ergodic theory and Property (T)	264
6.4	Uniqueness of invariant means	276
6.5	Exercises	279
7	A short list of open questions	282

PART II: BACKGROUND ON UNITARY REPRESENTATIONS	287
A Unitary group representations	289
A.1 Unitary representations	289
A.2 Schur's Lemma	296
A.3 The Haar measure of a locally compact group	299
A.4 The regular representation of a locally compact group	305
A.5 Representations of compact groups	306
A.6 Unitary representations associated to group actions	307
A.7 Group actions associated to orthogonal representations	311
A.8 Exercises	321
B Measures on homogeneous spaces	324
B.1 Invariant measures	324
B.2 Lattices in locally compact groups	332
B.3 Exercises	337
C Functions of positive type and GNS construction	340
C.1 Kernels of positive type	340
C.2 Kernels conditionally of negative type	345
C.3 Schoenberg's Theorem	349
C.4 Functions on groups	351
C.5 The cone of functions of positive type	357
C.6 Exercises	365
D Unitary Representations of locally compact abelian groups	369
D.1 The Fourier transform	369
D.2 Bochner's Theorem	372
D.3 Unitary representations of locally compact abelian groups	373
D.4 Local fields	377
D.5 Exercises	380
E Induced representations	383
E.1 Definition of induced representations	383
E.2 Some properties of induced representations	389
E.3 Induced representations with invariant vectors	391
E.4 Exercises	393

F	Weak containment and Fell's topology	395
F.1	Weak containment of unitary representations	395
F.2	Fell topology on sets of unitary representations	402
F.3	Continuity of operations	407
F.4	The C^* -algebras of a locally compact group	411
F.5	Direct integrals of unitary representations	413
F.6	Exercises	417
G	Amenability	420
G.1	Invariant means	421
G.2	Examples of amenable groups	424
G.3	Weak containment and amenability	427
G.4	Kesten's characterisation of amenability	433
G.5	Følner's property	440
G.6	Exercises	445
	<i>Bibliography</i>	449
	<i>Index</i>	468