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Cambridge University Press
978-0-521-88718-2 - A Primer of Infinitesimal Analysis, Second Edition
John L. Bell
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A PRIMER OF INFINITESIMAL ANALYSIS
SECOND EDITION

One of the most remarkable recent occurrences in mathematics is the refounding, on a rigorous basis, of the idea of infinitesimal quantity, a notion that played an important role in the early development of calculus and mathematical analysis. In this new edition, basic calculus, together with some of its applications to simple physical problems, is represented through the use of a straightforward, rigorous, axiomatically formulated concept of 'zero-square', or 'nilpotent' infinitesimal – that is, a quantity so small that its square and all higher powers can be set, literally, to zero. The systematic employment of these infinitesimals reduces the differential calculus to simple algebra and, at the same time, restores to use the 'infinitesimal' methods figuring in traditional applications of the calculus to physical problems – a number of which are discussed in this book. This edition also contains some additional applications to physics.

John L. Bell is Professor of Philosophy at the University of Western Ontario. He is the author of seven other books, including *Models and Ultraproducts* with A. B. Slomson, *A Course in Mathematical Logic* with M. Machover, *Logical Options* with D. DeVidi and G. Solomon, *Set Theory: Boolean-Valued Models and Independence Proofs*, and *The Continuous and the Infinitesimal in Mathematics and Philosophy.*

'This might turn out to be a boring, shallow book review: I merely LOVED the book. . . the explanations are so clear, so considerate; the author must have taught the subject many times, since he anticipates virtually every potential question, concern, and misconception in a student's or reader's mind.'

– Marion Cohen, *MAA Reviews*

'The book will be of interest to philosophically orientated mathematicians and logicians.'

– *European Mathematical Society*

'John Bell has done a first rate job in presenting an elementary introduction to this fascinating subject. . . . I recommend it highly.'

– J. P. Mayberry, *British Journal for the Philosophy of Science*

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Preface

A remarkable recent development in mathematics is the refounding, on a rigorous basis, of the idea of *infinitesimal quantity*, a notion which, before being supplanted in the nineteenth century by the limit concept, played a seminal role within the calculus and mathematical analysis. One of the most useful concepts of infinitesimal to have thus acquired rigorous status is that of a quantity so small (but not actually zero) that its square and all higher powers can be set to zero. The introduction of these 'zero-square' or 'nilpotent' infinitesimals opens the way to a revival of the intuitive, and remarkably efficient, 'pre-limit' approaches to the calculus: this little book is an attempt to get the process going at an elementary level. It begins with a historico-philosophical introduction in which the leading ideas of the basic framework – that of *smooth infinitesimal analysis* or analysis in *smooth worlds* – are outlined. The first chapter contains an axiomatic description of the essential technical features of smooth infinitesimal analysis. In the chapters that follow, nilpotent infinitesimals are used to develop single- and multi-variable calculus (with applications), the definite integral, and Taylor's theorem. The penultimate chapter contains a brief introduction to *synthetic differential geometry* – the transparent formulation of the differential geometry of manifolds made possible in smooth infinitesimal analysis by the presence of nilpotent infinitesimals. In the final chapter we outline the novel logical features of the framework. Scattered throughout the text are a number of straightforward exercises which the reader is encouraged to solve.

My purpose in writing this book has been to show how the traditional infinitesimal methods of mathematical analysis can be brought up to date – restored, so to speak – allowing their beauty and utility to be revealed anew. I believe that the greater part of its contents will be intelligible – and rewarding – to anyone with a basic knowledge of the calculus.*

* The only exception to this occurs in Chapters 7 and 8, and the Appendix (all of which can be omitted at a first reading) whose readers are assumed to have a slender acquaintance with differential geometry, logic, and category theory, respectively.

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A final remark: The theory of infinitesimals presented here should not be confused with that known as *nonstandard analysis*, invented by Abraham Robinson in the 1960s. The infinitesimals figuring in his formulation are '*invertible*' (arising, in fact, as the 'reciprocals' of *infinitely large* quantities), while those with which we shall be concerned, being nilpotent, cannot possess inverses. The two theories also have quite different mathematical origins, nonstandard analysis arising from developments in logic, and that presented here from category theory. For a brief discussion of nonstandard analysis, see the final chapter of the book.

In this second edition of the book, I have added some new material and taken the opportunity to correct a number of errors.

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I am also grateful to Thomas Streicher for his careful reading of the first edition and for pointing out a number of errors.

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