

## Index

- accelerated particle, 41
- acceleration, 46–48
  - absolute, 2
  - of the universe, 351–353
- accretion disk, 317
- active gravitational mass, 197, 202, 355
- adiabatic, 103
- affine parameter, 161, 166, 175
- angular diameter distance, 350–351
- angular momentum, 180, 287
- angular velocity, 180, 287, 310
- antisymmetric metric, 75
- area coordinate, 257
- area theorem of Hawking, 307–308
- Assumption of Mediocrity, 341
- asymptotically flat spacetime, 195, 259
- Auriga bar, 219
  
- background Lorentz transformations, 190, 238
- background Minkowski spacetime, 205
- bar detectors, 214, 218–219
- baryons, 100, 359, 363
- basis one-forms, 60–61
  - polar coordinate, 122–123
- basis vectors, 36–37, 60, 375
  - derivatives of, 125–126
  - inverse transformations, 39–41
  - orthonormal, 375
  - polar coordinate, 122–123, 135
  - transformation of, 37–39
- beam detectors, 215, 222–224
- Bianchi identities, 163–165, 166, 188, 262
- Big Bang, 356, 365, 368
  - gravitational waves, 213, 246–247
- Big Crunch, 356
- binary pulsar, 195, 291
- binary star system, 231, 232, 242–244, 275
- Birkhoff's theorem, 263
- black holes, 27, 258, 304–318, 364
  - degrees of freedom, 318
  - dynamical, 322–323
  - formation of, 269–271, 304–307
  - formula for the radius of, 282
  - general properties, 307–309
  - intermediate-mass, 321–322
  - Kerr, 307, 309–310, 316
  - merger of, 243
  - metric, 269
  - in Newtonian gravity, 281–282
  - quantum mechanical emission of radiation, 323–327
  - real, 318–323
  - Schwarzschild, 307, 326
  - of stellar mass, 269–271, 319–320
  - supermassive, 320–321
- Boyer–Lindquist coordinates, 309
- branes, 368, 369
- Buchdahl's interior solution, 267–269
- Buchdahl's theorem, 269
  
- center of momentum (CM) frame, 44
- Chandrasekhar limit, 274
- Chandrasekhar, S., 186, 318
- chirping signals, 243, 244
- Christoffel symbols, 127, 138, 157, 162, 174, 176, 178, 207
  - calculation from the metric, 133–134
  - and the metric, 131–135
- circularly polarized radiation, 232
- clusters, 336
- cold dark matter model, 363
- cold electron gas, 272, 273
- column vectors, 375
- comma-goes-to-semicolon rule, 174
- components
  - of the metric, 75
  - of one-forms, 66–67
- connecting vector, 162
- conservation
  - of energy, 99
  - of energy-momentum, 98–99
  - of entropy, 175
  - of four-momentum, 43–44, 175
  - of particles, 100, 174
  - of quantities, 178–180, 283–284
- conservation laws, 102–104
- constants, in SI and geometrized units, 187t
- continuum, 84
- contraction, 59
- conventions, sign of interval, 18–19
- cooling, 363
- coordinate singularities, 298–299

- coordinate systems, 300–301
- coordinates
  - Kruskal–Szekeres, 301–304
  - for spherically symmetric spacetimes, 256–258
- Copernican Principle, 339, 341
- cosmic censorship conjecture, 308
- cosmic microwave background radiation, 246, 351, 361–363
- cosmic microwave background temperature
  - perturbations, 214
- cosmography, 348–351
- cosmological constant, 188, 352, 354, 357, 358
- cosmological distance ladder, 349
- cosmological expansion factor, 353
- cosmological principle, 340–341, 356
- cosmological redshift, 345–348
- cosmological singularity, 357
- cosmology, 246, 335–373
  - matter-dominated, 354
  - metrics, 341–344
  - radiation-dominated, 354
- covariant derivative, 127–128, 130, 151, 161, 166
- covariant differentiation, 150–153
- covariant vectors. *see* one-forms
- covectors. *see* one-forms
- critical energy density, 358–361
- curvature, 111–141, 142
  - intrinsic and extrinsic, 153–154
  - relation of gravitation to, 111–118
  - role of, 117–118
  - of a sphere, 154
- curvature coordinate, 257
- curvature singularity, 300
- curvature tensor, 157–163
- curved manifolds, 142–170
- curved spacetime
  - two-spheres in, 256–257
  - waves in, 212
- curves
  - definition, 120–121
  - lengths and volumes of, 147–148
  - and vectors, 120–122
  - cylinder, intrinsic geometry of, 153, 154
- D’Alembertian, 193
- dark energy, 337, 355, 356, 360
- dark energy density, 358, 361
- dark matter, 296–297, 337, 359, 364
- dark pressure, 355
- deceleration parameter, 347
- decoupling, 361–363
  - universe after, 363–364
- derivatives
  - of basis vectors, 125–126
  - covariant, 127–128, 130, 151, 161, 166
  - of general vectors, 126–127
  - notation for, 65
  - of one-forms, 129–131
  - of tensors, 76, 129–131
  - vector field, 150
- determinant, 376, 377
- differentiable manifolds, 143
- differential structure, 143
- dispersion relation, of a wave, 204
- displacement vector, 33
- distances, measuring with light, 220–222
- divergence, 129
- divergence formula, 152–153
- Doppler-shift formula for photons, 49
- dot product, 57, 123
- drag-free operation, 226
- dragging of inertial frames, 310–311
- dual vector space, 59
- dummy index, 35, 37
- dust, 85–88
  - stress-energy tensor, 91–93
- dynamical black holes, 322–323
- Earth, mass of, 187
- Einstein, Albert, 2
- Einstein equations, 184–202
  - integration of, 264–265
  - purpose and justification of, 184–187
  - static perfect fluid, 260–262
  - in vacuum, 203
  - for weak gravitational fields, 189–194
- Einstein equivalence principle, 115, 173, 174, 185
- Einstein law of composition of velocities, 23
- Einstein summation convention, 34
- Einstein tensor, 164–165, 166, 193, 260, 343
- electromagnetic radiation, 213
- electromagnetic spectrum, 213
- electromagnetic waves, 212
- electromagnetism, 311
- electrons, 272
- element
  - of a continuum, 84
  - of proper volume, 166
- ellipse, equation of, 289
- elsewhere, 14
  - absolute, 13
- energy
  - density of, 91–92, 94t
  - and gravitational waves, 234–242
  - at infinity, 259
  - law of conservation of, 99
  - lost by a radiating system, 239–242
  - of a particle, 48
- energy flux, 98
  - gravitational waves, 235–239
- energy-momentum, conservation of, 98–99
- entropy, 95, 96
  - law of conservation of, 175
- equation of geodesic deviation, 163

- equations of motion, 261  
 equations of state, 261  
   neutron stars, 275  
 equivalence, principle of, 114–115  
 ergoregion, 311–312, 316  
 ergosphere, 312  
 ergotoroids, 312  
 escape velocity, 281  
 Euclidean space, 142, 156  
 event horizon, 304, 306  
 events, 5, 6, 12–13  
 exact plane wave, 210–212  
 extremal length, 157  
 extrinsic curvature, 153, 154
- far field, of stationary relativistic sources,  
   195–196  
 Fermi momentum, 272, 273  
 Fermi particles, 274  
 field equations  
   of GR, 185  
   of linearized theory, 194  
 first law of thermodynamics, 94–96  
 flat manifold, 160, 166  
 flat space, spherical coordinates, 256  
 fluid element, 97  
   four-velocity, 94t  
 fluids, 84–85, 93–100  
 flux, across a surface, 85–87, 90  
 flux vector, 94t  
 four-acceleration, 104  
 four-momentum, 42–44  
   conservation of, 43–44, 175  
   of a particle, 49  
 four-vector, 35n  
 four-velocity, 41–42, 46–48, 49, 62, 87  
   of fluid element, 94t  
 Fourier transform, 244  
 free index, 35  
 free particles, effect of waves on, 206–207  
 freely falling frame, 172  
 freely falling particle, 179  
 frequency, of a wave, 204  
 function, 57–58  
 future, 14  
   absolute, 13
- GAIA satellite, 364  
 galactic bulge, 321  
 galaxies, 336  
   formation of, 321, 363–364  
 Galilean law of addition of velocities, 2, 23  
 gauge transformations, 191  
 gauge, transverse-traceless, 204–206  
 Gauss' law, 105–106, 153  
 general fluids, 93–100  
 general horizon, 304
- general relativity, 3, 104–105, 194, 322, 335, 351  
   cosmology and, 336  
   curved spacetime of, 76  
   field equations of. *see* Einstein equations  
 general stress-energy tensor, 96  
 general vector, 35  
 GEO600, 226  
 geodesic deviation, 161–163  
 geodesic equation, 179  
 geodesics, 156–157, 166  
   extremal length, 157  
 geometer, 74  
 geometrical optics, 212  
 GPS navigation system, 113  
 GR. *see* general relativity  
 gradient, 62–65, 90, 128  
 gravitational collapse, 245  
 gravitational fields, 184, 194–197  
 gravitational lensing, 212, 296–298  
 gravitational potential energy, 178  
 gravitational radiation, 195, 203–255  
 gravitational redshift experiment, 112–113, 115  
 gravitational self-energy, 178  
 gravitational spacetime, 117  
 gravitational wave detectors, 207  
 gravitational wave forces, 207–208  
 gravitational wave spectrum, 213  
 gravitational waves, 194, 212  
   amplitude of, 227  
   astrophysical sources of, 242–247  
   from the Big Bang, 246–247  
   detection of, 213–227  
   energy and, 234–242  
   energy flux, 235–239  
   generation of, 227–234  
   polarization of, 210  
   propagation of, 203–212  
 gravitomagnetism, 311  
 graviton, 50  
 gravity, 114, 172, 173  
   deflection of light by, 293–295  
   Newtonian, 194–195, 336
- harmonic gauge, 193  
 harmonic oscillator, 215, 216, 234  
 Hawking area theorem, 307–308  
 Hawking process, 323–327  
 heat conduction, 100–101  
 Higgs particle, 366  
 Hilbert space, 71  
 homogeneity, 367  
 horizon, 300, 303, 304, 306  
   Kerr, 312–313  
   optical, 341  
   particle, 340  
 Hubble constant, 339, 347, 359  
 Hubble flow, 338

- Hubble parameter, 339, 346, 347  
 Hubble time, 339  
 Hulse–Taylor binary pulsar, 240, 291  
 hydrogen, ionization of, 362  
 hyperbola  
   invariant, 14–17  
   tangent to, 17  
 hypersurface, 44
- impact parameter, 293  
 index, 191  
   contravariant, 73  
   covariant, 73  
   dummy, 35, 37  
   free, 35  
   Greek, 6  
   Latin, 6  
   raising and lowering of, 74–76  
 inertial frames, 4, 91, 114, 178, 205  
   dragging of, 310–311  
   freely falling, 116  
   local, 116–117  
   nonexistence at rest on Earth, 113–114  
 inertial observers, 2, 3–4  
 inflation, 367, 368  
 inner product, 374–375  
 interferometers, 214, 219, 222–224  
   observations, 224–227  
 intermediate-mass black holes, 271, 321–322  
 internal energy per particle, 94t  
 interval  
   invariance of, 9–14, 22  
   spacelike, 111  
   timelike, 111  
 intrinsic curvature, 153–154  
 invariant hyperbolae, 14–17  
 inverse transformations, 59  
 isotropic coordinates, 292
- Jacobian, 118, 147
- Keplerian motion of the planets, 177  
 Kerr black hole, 243, 307, 309–310, 316  
 Kerr horizon, 308, 312–313  
 Kerr metric, 310, 312, 327  
   equatorial photon motion in, 313–316  
 Kerr, Roy, 318  
 Kerr solution, 307  
 Kronecker delta, 10, 36, 40, 45, 75, 376  
 Kruskal–Szekeres coordinates, 301–304
- Laplace, Pierre, 281  
 Laplacian, 129  
 Large Hadron Collider, 366  
 laser interferometer gravitational wave detector, 210,  
   214, 219, 220  
 law of conservation of energy, 99  
   law of conservation of entropy, 175  
   law of conservation of particles, 174  
 lengths of curves, 147  
 Lense–Thirring effect, 311  
 lensing, gravitational, 212, 296–298  
 light  
   gravitational deflection of, 293–295  
   measuring distances with, 220–222  
 light cones, 12f, 13, 300–301, 302, 303  
 LIGO detectors, 224, 225f, 226, 243, 246  
 line element, 124, 342  
   of flat spacetime, 211  
 linear algebra, 374–377  
 linearity, 57, 143  
 linearized theory, field equations of, 194  
 LISA, 214, 226, 227, 242, 243, 246, 321, 360, 364  
 local flatness, 145–146, 151  
 local flatness theorem, 149–150  
 local inertial frames, 116–117, 146, 216  
 locally trapped surfaces, 306  
 loop quantum gravity, 368  
 Lorentz contraction, 1, 18, 85, 86f  
 Lorentz frame. *see* inertial frames  
 Lorentz gauge, 193–194  
 Lorentz transformation, 1, 21–22, 34, 39, 190  
 luminosity, 240, 350  
 luminosity distance, 349
- macroscopic quantities, definition of, 94  
 magnetism, 311  
 magnification, 296  
 magnitude–redshift diagram, 351  
 main sequence stars, 270  
 manifolds  
   coordinates, 142  
   definition, 142  
   differentiable, 143  
   dimension, 142  
   flat, 160  
   Riemannian, 144–150, 165  
 mass  
   of the Earth, 187  
   of a relativistic body, 196–197  
   of a star, 268  
 mass density, 184  
 mass function, 262  
 matrices, 375–377  
   determinant, 376, 377  
   dimension of, 375  
   inverse, 377  
   multiplication, 376  
   vector space, 376  
 matter-dominated cosmology, 354  
 matter energy density, 361  
 MCRF, 41, 42, 91, 94, 96, 101, 103, 174, 184  
   symmetry in, 97–98  
 Mercury, perihelion of, 288

- metric, 56, 64, 66, 111, 123–124, 165, 171, 185, 341
  - antisymmetric, 75
  - black hole, 269
  - calculation of Christoffel symbols from, 133–134
  - Christoffel symbols and, 131–135
  - components of, 58
  - cosmological, 341–344
  - definition, 57
  - of Euclidean space, 133
  - inverse, 69–70, 124
  - and local flatness, 145–146
  - as a mapping of vectors into one-forms, 68–72
  - Minkowski, 177
  - mixed components of, 75
  - signature of, 145
  - static spherically symmetric spacetimes, 258–259
  - symmetric, 68
  - of waves, 221
- Michell, John, 281
- microlensing, 297
- Minkowski metric, 177
- Minkowski space, 117, 256, 345
- momentarily comoving reference frame. *see* MCRF
- momentum density, 98
- momentum, Fermi, 272, 273
- momentum, flux of, 96–97
- motion
  - equation of, 261
  - of a freely falling particle, 175
- naked singularities, 308
- nearly Lorentz coordinate systems, 189–190
- neutrinos, 50, 297
- neutron stars, 243, 244–245, 271, 274–275, 282, 319
  - forces which support, 271–273
- neutrons, 100, 275
- Newtonian dark star, 282
- Newtonian gravitational fields, 184, 194–197
- Newtonian gravity, 194–195, 336
  - black holes in, 281–282
- Newtonian limit, 188, 194–195
- Newtonian potential, 195, 196, 232, 240
- Newtonian stars
  - equation of hydrostatic equilibrium, 266
  - structure of, 265–266
- Newtonian tidal gravitational force, 233
- Newton's laws, 2
- noncoordinate bases, 135–138
- normal one-forms, 66, 72
- normal vector, 66, 72
- notation, for derivatives, 65
- notation, spacetime diagrams, 6
- null rays, 304–305, 306
- null vector, 45
- number density, 85, 94t
  - as a timelike flux, 88–89
- number-flux four-vector, 87–88
- observation, 4
- Olbers' Paradox, 337
- one-forms, 58–66, 70–71
  - basis, 60–61
  - defining a surface, 89–90
  - as derivative of a function, 62–65
  - general properties, 58–60
  - image of, 61–62
  - inner product, 72
  - linearity, 143
  - magnitude of, 71–72
  - normal, 66, 72
  - polar coordinate basis, 122–123
  - representation of a frame by, 91
  - and surfaces, 88–91
  - vectors and, 119–120
- optical horizon, 341
- orbits, 284–287
  - circular, 286
  - hyperbolic, 285
- order-of-magnitude estimates, 232–233
- orthogonality, 45, 46, 303
- orthonormality, 45
- paradoxes, 19
  - and physical intuition, 23–24
  - pole-in-the-barn, 23
  - space-war, 23
  - twin paradox, 21, 23, 25–27
- parallel-transport, 155–156, 158, 166
- parallelepiped, 87
- parallelism, 117
- parameters, affine, 157, 161, 166, 175
- particle horizon, 340
- particles
  - angular momentum, 283
  - conservation of, 100
  - energy of, 48
  - four-momentum of, 49
  - infalling, 299–300
  - law of conservation of, 174
  - potential, 284
- past, the, 14
  - absolute, 13
- path, 120
- Penrose process, 316–318, 325
- perfect fluids, 84–85, 100–104, 353
- periastron shift, 291
- pericenter shift, 291, 292
- perihelion shift, 287–290
- period, 287
- photons, 49–50
  - angular momentum, 283

- motion in the equatorial plane, 313–316
  - potential, 284
  - zero rest-mass, 49–50
- Planck mass, 326
- Planck time, 357
- plane-fronted waves with parallel rays, 212
- plane wave, 204, 206
  - exact, 210–212
- planetary nebulae, 270
- planets, Newtonian (Keplerian) motion of, 293
- plasma fluctuations, 222
- polar coordinate basis vectors, 135
- polar coordinates
  - tensor algebra in, 118–124
  - tensor calculus in, 125–131
- polar unit basis, 136–137
- polarization, of gravitational waves, 210
- polytrope, 268
- Population III stars, 364
- post-Newtonian effects, 195, 202
- post-Newtonian gravity, 291–293
- Pound–Rebka–Snider experiment, 112, 113, 114, 115
- precession, 288–289
- pressure, 94t, 102, 104
  - quantum mechanical, 271–273
- pressure gradient, 105
- principle of equivalence, 114–115
- principle of relativity, 1, 19, 20
- proof masses, 226
- proper distance, 206, 348–349
- proper length, 18f, 208
- proper radial distance, 259
- proper time, 17
- proper volume element, 148, 265
- protons, 100
- pseudo-Riemannian space, 75
- pulsar timing, 214, 215, 246
- pulsars, 232, 244, 275
  
- quadrupole approximation for gravitational radiation, 230
- quadrupole moment tensor, 229
- quality factor of an oscillator, 218
- quantities, conservation of, 178–180, 283–284
- quantum nondemolition detectors, 247
- quasar jets, 317
- quasars, 321
  
- r-mode instability, 244
- radar, 220
- radiating system, energy lost by, 239–242
- radiation-dominated cosmology, 354
- real black holes, 318–323
- realistic stars and gravitational collapse, 269–275
- recombination, *see* decoupling
- red giant, 270
  
- redshift, 350, 362
  - cosmological, 345–348
  - gravitational, 112–113, 115
  - of photons, 259
- relativistic body, mass of, 196–197
- relativity, principle of, 1, 19, 20
- resonant mass detectors, 214, 215–218
- rest mass, 42, 43
- rest-mass density, 94t
- Ricci scalar, 164, 166
- Ricci scalar curvature, 342
- Ricci tensor, 164, 166, 185
- Riemann curvature tensor, 159
- Riemann tensor, 139, 161, 166, 174, 192, 208
- Riemannian manifolds, 144–150, 165
- Riemannian spaces, 75, 117
- Robertson–Walker metric, 343
  - closed, 344
  - flat, 344
  - hyperbolic, 345
  - open, 345
  - spherical, 344
  
- scalar, covariant derivative of, 128
- scalar product, 44–46, 56, 374–375
- Schwarzschild black hole, 307, 326
- Schwarzschild constant-density interior solution, 266–267
- Schwarzschild geometry, 281
- Schwarzschild horizon, 300, 304, 305
- Schwarzschild metric, 262–263, 292, 323, 327
- Schwarzschild spacetime, trajectories in, 281–298
- second law of thermodynamics, 96
- shot noise, 225
- SI units, 4–5, 186
- signature of the metric, 145
- simultaneity, 172
  - failure of, 8
- singularity
  - coordinate, 298–299
  - cosmological, 357
  - curvature, 300, 308
  - naked, 308
- slow-motion assumption, 227
- Solar System, post-Newtonian effects, 195
- space, measuring the stretching of, 209–210
- spacecraft tracking, 214, 222
- spacelike interval, 111
- spacetime, 1, 171–172
  - asymptotically flat, 195, 259
  - curvature, 154
  - metric of, 171
  - quantizing, 368
  - slightly curved, 175–177
  - spherically symmetric, 258
  - static spherically symmetric, 258–260

- spacetime diagrams, 5–6  
   construction of coordinates, 12f  
   notation, 6
- special relativity, 2, 178  
   fundamental principles of, 1–3  
   Lorentz transformation, 190  
   metric of, 111  
   perfect fluids in, 84–110
- specific entropy, 94t  
   of the universe, 365
- speed of light, universality of, 1, 2, 10, 21
- sphere, curvature, 154–155
- spherical solutions for stars, 256–280
- spherical star, collapse to form a black hole, 305
- spherical symmetric spacetimes, coordinates for, 256–258
- spinning neutron stars, 244–245
- spinor space, 75
- Square Kilometer Array (SKA) radio telescope, 214
- SQUIDS, 219
- SR. *see* special relativity
- standard candle, 349
- standard sirens, 244
- stars  
   characteristics of, 309  
   collapse of spherical star, 305  
   composition of, 271  
   exact interior solutions, 266  
   exterior geometry, 262–263  
   interior structure, 263–266  
   mass of, 265, 268  
   population III, 364  
   realistic stars and gravitational collapse, 269–275  
   spherical solutions for, 256–280  
   surface of, 264
- static limit, 312
- static spacetimes  
   definition, 258  
   spherically symmetric, 258–260
- stationary horizons, 307
- stationary relativistic sources, far field of, 195–196
- stellar mass black holes, 269–271, 319–320
- stress-energy tensor, 92–93, 101, 260, 353
- stretching of space, 209–210
- string theory, 368–369
- summation convention, 37
- supermassive black holes, 320–321
- supernova, 245
- supernova explosions, 320, 352
- supersymmetry, 366
- surface element, 90
- surfaces  
   flux across, 90  
   one-forms and, 88–91
- symmetry, of one-forms, 67–68
- tangent vector, 121
- temperature, 94t
- tension, 356
- tensor algebra  
   in polar coordinates, 118–124  
   review, 143–144
- tensor analysis, 56–83
- tensor calculus, in polar coordinates, 125–131
- tensors, 143  
   antisymmetric, 68  
   components of, 58, 66–67  
   covectors, 58–66  
   definition, 56–57  
   differentiation of, 76  
   field, 143  
   general stress-energy, 96  
   linear function of  $M$  one-forms and  $N$  vectors into real numbers, 73  
   linear function of  $M$  one-forms into real numbers, 73  
   metric, 123–124  
   permissible tensor operations, 144  
   reduced quadrupole moment, 230  
   Riemannian, 139, 161, 166, 174, 192, 208  
   symmetric, 67  
   trace-free moment, 230  
   with two vector arguments, 66
- thermal noise, 218, 224
- thermodynamics  
   first law of, 94–96  
   second law, 96
- three-dimensional wave equation, 203
- tidal forces, 117, 173, 207–208, 209, 233
- time dilation, 17
- timelike interval, 111
- Tolman–Oppenheimer–Volkov equation, 263–264  
   integration of, 264–265
- transducer, 219
- transponders, 222
- transverse-traceless gauge, 204–206
- turning point, 285
- twin paradox, 23, 25–27
- two-spheres  
   in a curved spacetime, 256–257  
   meshing into a three-space, 257
- uncertainty principle of quantum mechanics, 308, 323
- unit vectors, 136
- units, 4–5  
   geometrized, 186–187  
   natural, 4  
   SI, 4–5, 186
- universality of the speed of light, 1, 2, 10, 21
- universe  
   acceleration of, 351–353  
   age of, 339  
   anisotropy, 339

- closed, 356
- after decoupling, 363–364
- evolution of, 361–369
- expansion of, 337, 338, 356
- flat, 356
- history of, 356
- homogeneity, 336, 337, 338–340
- hyperbolic, 356
- isotropic, 336, 338–340
- measures of distance in, 348–351
- models of, 340–341
- radiation-dominated, 362, 365
- re-collapsing, 356
- Robertson–Walker, 353–358
- scale factor, 347
- specific entropy, 365
  
- vacuum energy, 367
- vector algebra, 36–41
  - metric and nonmetric, 75–76
- vector analysis, 33–55
- vector field, 128
- vector space, 374–375
  - dimension, 376
  - dual, 59
  - matrices, 376
- vectors, 70–71
  - basis, 375
  - column, 375
  - connecting, 162
  - curves and, 120–122
  - definition, 33–36
  - derivatives of general, 126–127
  - displacement, 33
  - divergence of, 137, 152
  - flux, 94t
  - as a function of one-forms, 72–73
  - gradient, 64
  - linearity, 143
  - linearly independent, 374
  - magnitude, 44
  - norm of, 375
  - normal, 66, 72
  - null, 44, 45
    - and one-forms, 119–120
  - orthogonal, 45, 46
  - polar coordinate basis, 122–123
  - scalar product, 44–46
  - spacelike, 44
  - tangent, 121
  - timelike, 44
  - unit, 136
- velocities, Galilean law of addition of, 2, 23
- velocity, absolute, 2
- velocity-composition law, 22–23
- vibrations, interferometers and, 224
- VIRGO detector, 224, 226
- viscosity, 101
- visual observation, 4
- volume element, 148
- volumes of curves, 147–148
  
- wave equation
  - exact solution, 233–234
  - three-dimensional, 203
- wave-function, 71
- wave generation, 227–232
- wave operator, 193
- waves
  - in a curved spacetime, 212
  - effect on free particles, 206–207
  - metric of, 221
- weak equivalence principle, 115, 172–173, 281
- weak-field Einstein equations, 192–194
- weak gravitational fields, 189–194
- Weber, Joseph, 218, 245
- white dwarf, 270, 273–274
  - forces which support, 271–273
- world line, 5
  
- zero angular-momentum observer (ZAMO), 315
- zero rest-mass particles, 49–50