

## VISCOELASTIC MATERIALS

Understanding viscoelasticity is pertinent to design applications as diverse as earplugs, gaskets, computer disks, satellite stability, medical diagnosis, injury prevention, vibration abatement, tire performance, sports, spacecraft, and music. This book fits a one-semester graduate course on the properties, analysis, and uses of viscoelastic materials. Those familiar with the author's precursor book, *Viscoelastic Solids*, will see that this book contains many updates and expanded coverage of the materials science, causes of viscoelastic behavior, properties of materials of biological origin, and applications of viscoelastic materials. The theoretical presentation includes both transient and dynamic aspects, with an emphasis on linear viscoelasticity to develop physical insight. Methods for the solution of stress analysis problems are developed and illustrated. Experimental methods for characterization of viscoelastic materials are explored in detail. Viscoelastic phenomena are described for a wide variety of materials, including viscoelastic composite materials. Applications of viscoelasticity and viscoelastic materials are illustrated with case studies.

Roderic Lakes is a Distinguished Professor in the Department of Engineering Physics at the University of Wisconsin–Madison. He is a Fellow in the American Association for the Advancement of Science (AAAS) and has been honored by becoming a Fellow in the American Society of Mechanical Engineers (ASME). He has won numerous teaching awards and is the author and coauthor of more than 194 archival publications; three books, including *Viscoelastic Solids* and *Biomaterials* (2nd and 3rd editions, with J. B. Park); and fourteen book chapters. The author's articles in *Science* and *Nature* are of particular note because they have led to numerous synergistic publications in a variety of disciplines.

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Roderic Lakes

*University of Wisconsin–Madison*



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In ancient rivalry with fellow spheres the sun still sings its  
glorious song and completes with tread of thunder the journey  
it has been assigned.

*Goethe*

## Contents

<i>Preface</i>	<i>page xvii</i>
<b>1 Introduction: Phenomena</b> . . . . .	<b>1</b>
1.1 Viscoelastic Phenomena	1
1.2 Motivations for Studying Viscoelasticity	3
1.3 Transient Properties: Creep and Relaxation	3
1.3.1 Viscoelastic Functions $J(t)$ , $E(t)$	3
1.3.2 Solids and Liquids	7
1.4 Dynamic Response to Sinusoidal Load: $E^*$ , $\tan\delta$	8
1.5 Demonstration of Viscoelastic Behavior	10
1.6 Historical Aspects	10
1.7 Summary	11
1.8 Examples	11
1.9 Problems	12
Bibliography	12
<b>2 Constitutive Relations</b> . . . . .	<b>14</b>
2.1 Introduction	14
2.2 Prediction of the Response of Linearly Viscoelastic Materials	14
2.2.1 Prediction of Recovery from Relaxation $E(t)$	14
2.2.2 Prediction of Response to Arbitrary Strain History	15
2.3 Restrictions on the Viscoelastic Functions	17
2.3.1 Roles of Energy and Passivity	17
2.3.2 Fading Memory	18
2.4 Relation between Creep and Relaxation	19
2.4.1 Analysis by Laplace Transforms: $J(t) \leftrightarrow E(t)$	19
2.4.2 Analysis by Direct Construction: $J(t) \leftrightarrow E(t)$	20
2.5 Stress versus Strain for Constant Strain Rate	20
2.6 Particular Creep and Relaxation Functions	21
2.6.1 Exponentials and Mechanical Models	21

2.6.2	Exponentials and Internal Causal Variables	26
2.6.3	Fractional Derivatives	27
2.6.4	Power-Law Behavior	28
2.6.5	Stretched Exponential	29
2.6.6	Logarithmic Creep; Kuhn Model	29
2.6.7	Distinguishing among Viscoelastic Functions	30
2.7	Effect of Temperature	30
2.8	Three-Dimensional Linear Constitutive Equation	33
2.9	Aging Materials	35
2.10	Dielectric and Other Forms of Relaxation	35
2.11	Adaptive and “Smart” Materials	36
2.12	Effect of Nonlinearity	37
2.12.1	Constitutive Equations	37
2.12.2	Creep–Relaxation Interrelation: Nonlinear	40
2.13	Summary	43
2.14	Examples	43
2.15	Problems	51
	Bibliography	52
<b>3</b>	<b>Dynamic Behavior . . . . .</b>	<b>55</b>
3.1	Introduction and Rationale	55
3.2	The Linear Dynamic Response Functions $E^*$ , $\tan\delta$	56
3.2.1	Response to Sinusoidal Input	57
3.2.2	Dynamic Stress–Strain Relation	59
3.2.3	Standard Linear Solid	62
3.3	Kramers–Kronig Relations	63
3.4	Energy Storage and Dissipation	65
3.5	Resonance of Structural Members	67
3.5.1	Resonance, Lumped System	67
3.5.2	Resonance, Distributed System	71
3.6	Decay of Resonant Vibration	74
3.7	Wave Propagation and Attenuation	77
3.8	Measures of Damping	79
3.9	Nonlinear Materials	79
3.10	Summary	81
3.11	Examples	81
3.12	Problems	88
	Bibliography	89
<b>4</b>	<b>Conceptual Structure of Linear Viscoelasticity . . . . .</b>	<b>91</b>
4.1	Introduction	91
4.2	Spectra in Linear Viscoelasticity	92
4.2.1	Definitions $H(\tau)$ , $L(\tau)$ and Exact Interrelations	92
4.2.2	Particular Spectra	93

## Contents

ix

4.3	Approximate Interrelations of Viscoelastic Functions	95
4.3.1	Interrelations Involving the Spectra	95
4.3.2	Interrelations Involving Measurable Functions	98
4.3.3	Summary, Approximate Relations	101
4.4	Conceptual Organization of the Viscoelastic Functions	101
4.5	Summary	104
4.6	Examples	104
4.7	Problems	109
	Bibliography	109
<b>5</b>	<b>Viscoelastic Stress and Deformation Analysis . . . . .</b>	<b>111</b>
5.1	Introduction	111
5.2	Three-Dimensional Constitutive Equation	111
5.3	Pure Bending by Direct Construction	112
5.4	Correspondence Principle	114
5.5	Pure Bending by Correspondence	116
5.6	Correspondence Principle in Three Dimensions	116
5.6.1	Constitutive Equations	116
5.6.2	Rigid Indenter on a Semi-Infinite Solid	117
5.6.3	Viscoelastic Rod Held at Constant Extension	119
5.6.4	Stress Concentration	119
5.6.5	Saint Venant's Principle	120
5.7	Poisson's Ratio $\nu(t)$	121
5.7.1	Relaxation in Tension	121
5.7.2	Creep in Tension	123
5.8	Dynamic Problems: Effects of Inertia	124
5.8.1	Longitudinal Vibration and Waves in a Rod	124
5.8.2	Torsional Waves and Vibration in a Rod	125
5.8.3	Bending Waves and Vibration	128
5.8.4	Waves in Three Dimensions	129
5.9	Noncorrespondence Problems	131
5.9.1	Solution by Direct Construction: Example	131
5.9.2	A Generalized Correspondence Principle	132
5.9.3	Contact Problems	132
5.10	Bending in Nonlinear Viscoelasticity	133
5.11	Summary	134
5.12	Examples	134
5.13	Problems	142
	Bibliography	142
<b>6</b>	<b>Experimental Methods . . . . .</b>	<b>145</b>
6.1	Introduction and General Requirements	145
6.2	Creep	146
6.2.1	Creep: Simple Methods to Obtain $J(t)$	146

6.2.2	Effect of Risetime in Transient Tests	146
6.2.3	Creep in Anisotropic Media	148
6.2.4	Creep in Nonlinear Media	148
6.3	Inference of Moduli	150
6.3.1	Use of Analytical Solutions	150
6.3.2	Compression of a Block	151
6.4	Displacement and Strain Measurement	152
6.5	Force Measurement	156
6.6	Load Application	157
6.7	Environmental Control	157
6.8	Subresonant Dynamic Methods	158
6.8.1	Phase Determination	158
6.8.2	Nonlinear Materials	160
6.8.3	Rebound Test	161
6.9	Resonance Methods	161
6.9.1	General Principles	161
6.9.2	Particular Resonance Methods	163
6.9.3	Methods for Low-Loss or High-Loss Materials	166
6.9.4	Resonant Ultrasound Spectroscopy	168
6.10	Achieving a Wide Range of Time or Frequency	171
6.10.1	Rationale	171
6.10.2	Multiple Instruments and Long Creep	172
6.10.3	Time–Temperature Superposition	172
6.11	Test Instruments for Viscoelasticity	173
6.11.1	Servohydraulic Test Machines	173
6.11.2	A Relaxation Instrument	174
6.11.3	Driven Torsion Pendulum Devices	174
6.11.4	Commercial Viscoelastic Instrumentation	178
6.11.5	Instruments for a Wide Range of Time and Frequency	179
6.11.6	Fluctuation–Dissipation Relation	182
6.11.7	Mapping Properties by Indentation	183
6.12	Wave Methods	184
6.13	Summary	188
6.14	Examples	188
6.15	Problems	200
	Bibliography	201
<b>7</b>	<b>Viscoelastic Properties of Materials . . . . .</b>	<b>207</b>
7.1	Introduction	207
7.1.1	Rationale	207
7.1.2	Overview: Some Common Materials	207
7.2	Polymers	208
7.2.1	Shear and Extension in Amorphous Polymers	208
7.2.2	Bulk Relaxation in Amorphous Polymers	212



*Contents*

xi

7.2.3	Crystalline Polymers	213
7.2.4	Aging and other Relaxations	214
7.2.5	Piezoelectric Polymers	214
7.2.6	Asphalt	214
7.3	Metals	215
7.3.1	Linear Regime of Metals	215
7.3.2	Nonlinear Regime of Metals	217
7.3.3	High-Damping Metals and Alloys	219
7.3.4	Creep-Resistant Alloys	224
7.3.5	Semiconductors and Amorphous Elements	225
7.3.6	Semiconductors and Acoustic Amplification	226
7.3.7	Nanoscale Properties	226
7.4	Ceramics	227
7.4.1	Rocks	227
7.4.2	Concrete	229
7.4.3	Inorganic Glassy Materials	231
7.4.4	Ice	231
7.4.5	Piezoelectric Ceramics	232
7.5	Biological Composite Materials	233
7.5.1	Constitutive Equations	234
7.5.2	Hard Tissue: Bone	234
7.5.3	Collagen, Elastin, Proteoglycans	236
7.5.4	Ligament and Tendon	237
7.5.5	Muscle	240
7.5.6	Fat	243
7.5.7	Brain	243
7.5.8	Vocal Folds	244
7.5.9	Cartilage and Joints	244
7.5.10	Kidney and Liver	246
7.5.11	Uterus and Cervix	246
7.5.12	Arteries	247
7.5.13	Lung	248
7.5.14	The Ear	248
7.5.15	The Eye	249
7.5.16	Tissue Comparison	251
7.5.17	Plant Seeds	252
7.5.18	Wood	252
7.5.19	Soft Plant Tissue: Apple, Potato	253
7.6	Common Aspects	253
7.6.1	Temperature Dependence	253
7.6.2	High-Temperature Background	254
7.6.3	Negative Damping and Acoustic Emission	255
7.7	Summary	255
7.8	Examples	255

7.9 Problems	256
Bibliography	257
<b>8 Causal Mechanisms . . . . .</b>	<b>271</b>
8.1 Introduction	271
8.1.1 Rationale	271
8.1.2 Survey of Viscoelastic Mechanisms	271
8.1.3 Coupled Fields	273
8.2 Thermoelastic Relaxation	274
8.2.1 Thermoelasticity in One Dimension	274
8.2.2 Thermoelasticity in Three Dimensions	275
8.2.3 Thermoelastic Relaxation Kinetics	276
8.2.4 Heterogeneity and Thermoelastic Damping	278
8.2.5 Material Properties and Thermoelastic Damping	280
8.3 Relaxation by Stress-Induced Fluid Motion	280
8.3.1 Fluid Motion in One Dimension	280
8.3.2 Biot Theory: Fluid Motion in Three Dimensions	281
8.4 Relaxation by Molecular Rearrangement	286
8.4.1 Glassy Region	286
8.4.2 Transition Region	287
8.4.3 Rubbery Behavior	289
8.4.4 Crystalline Polymers	291
8.4.5 Biological Macromolecules	292
8.4.6 Polymers and Metals	292
8.5 Relaxation by Interface Motion	292
8.5.1 Grain Boundary Slip in Metals	292
8.5.2 Interface Motion in Composites	294
8.5.3 Structural Interface Motion	294
8.6 Relaxation Processes in Crystalline Materials	294
8.6.1 Snoek Relaxation: Interstitial Atoms	294
8.6.2 Zener Relaxation in Alloys: Pairs of Atoms	298
8.6.3 Gorsky Relaxation	299
8.6.4 Granato–Lücke Relaxation: Dislocations	300
8.6.5 Bordoni Relaxation: Dislocation Kinks	303
8.6.6 Relaxation Due to Phase Transformations	305
8.6.7 High-Temperature Background	314
8.6.8 Nonremovable Relaxations	315
8.6.9 Damping Due to Wave Scattering	316
8.7 Magnetic and Piezoelectric Materials	316
8.7.1 Relaxation in Magnetic Media	316
8.7.2 Relaxation in Piezoelectric Materials	318
8.8 Nonexponential Relaxation	322
8.9 Concepts for Material Design	323
8.9.1 Multiple Causes: Deformation Mechanism Maps	323

*Contents*

xiii

8.9.2 Damping Mechanisms in High-Loss Alloys	326
8.9.3 Creep Mechanisms in Creep-Resistant Alloys	326
8.10 Relaxation at Very Long Times	327
8.11 Summary	327
8.12 Examples	328
8.13 Problems and Questions	332
Bibliography	332
<b>9 Viscoelastic Composite Materials . . . . .</b>	<b>341</b>
9.1 Introduction	341
9.2 Composite Structures and Properties	341
9.2.1 Ideal Structures	341
9.2.2 Anisotropy due to Structure	342
9.3 Prediction of Elastic and Viscoelastic Properties	344
9.3.1 Basic Structures: Correspondence Solutions	344
9.3.2 Voigt Composite	345
9.3.3 Reuss Composite	345
9.3.4 Hashin–Shtrikman Composite	346
9.3.5 Spherical Particulate Inclusions	347
9.3.6 Fiber Inclusions	349
9.3.7 Platelet Inclusions	349
9.3.8 Stiffness-Loss Maps	350
9.4 Bounds on the Viscoelastic Properties	353
9.5 Extremal Composites	354
9.6 Biological Composite Materials	356
9.7 Poisson’s Ratio of Viscoelastic Composites	357
9.8 Particulate and Fibrous Composite Materials	358
9.8.1 Structure	358
9.8.2 Particulate Polymer Matrix Composites	359
9.8.3 Fibrous Polymer Matrix Composites	361
9.8.4 Metal–Matrix Composites	362
9.9 Cellular Solids	363
9.10 Piezoelectric Composites	366
9.11 Dispersion of Waves in Composites	366
9.12 Summary	367
9.13 Examples	367
9.14 Problems	370
Bibliography	370
<b>10 Applications and Case Studies . . . . .</b>	<b>377</b>
10.1 Introduction	377
10.2 A Viscoelastic Earplug: Use of Recovery	377
10.3 Creep and Relaxation of Materials and Structures	378
10.3.1 Concrete	378

10.3.2 Wood	378
10.3.3 Power Lines	379
10.3.4 Glass Sag: Flowing Window Panes	380
10.3.5 Indentation: Road Rutting	380
10.3.6 Leather	381
10.3.7 Creep-Resistant Alloys and Turbine Blades	381
10.3.8 Loosening of Bolts and Screws	382
10.3.9 Computer Disk Drive: Case Study of Relaxation	384
10.3.10 Earth, Rock, and Ice	385
10.3.11 Solder	386
10.3.12 Filaments in Light Bulbs and Other Devices	387
10.3.13 Tires: Flat-Spotting and Swelling	388
10.3.14 Cushions for Seats and Wheelchairs	388
10.3.15 Artificial Joints	389
10.3.16 Dental Fillings	389
10.3.17 Food Products	389
10.3.18 Seals and Gaskets	390
10.3.19 Relaxation in Musical Instrument Strings	390
10.3.20 Winding of Tape	391
10.4 Creep and Recovery in Human Tissue	391
10.4.1 Spinal Discs: Height Change	391
10.4.2 The Nose	392
10.4.3 Skin	392
10.4.4 The Head	393
10.5 Creep Damage and Creep Rupture	394
10.5.1 Vajont Slide	394
10.5.2 Collapse of a Tunnel Segment	394
10.6 Vibration Control and Waves	394
10.6.1 Analysis of Vibration Transmission	394
10.6.2 Resonant (Tuned) Damping	397
10.6.3 Rotating Equipment Vibration	397
10.6.4 Large Structure Vibration: Bridges and Buildings	398
10.6.5 Damping Layers for Plate and Beam Vibration	399
10.6.6 Structural Damping Materials	400
10.6.7 Piezoelectric Transducers	402
10.6.8 Aircraft Noise and Vibration	402
10.6.9 Solid Fuel Rocket Vibration	404
10.6.10 Sports Equipment Vibration	404
10.6.11 Seat Cushions and Automobiles: Protection of People	404
10.6.12 Vibration in Scientific Instruments	406
10.6.13 Waves	406
10.7 “Smart” Materials and Structures	407
10.7.1 “Smart” Materials	407
10.7.2 Shape Memory Materials	408

## Contents

xv

10.7.3 Self-Healing Materials	409
10.7.4 Piezoelectric Solid Damping	409
10.7.5 Active Vibration Control: “Smart” Structures	409
10.8 Rolling Friction	409
10.8.1 Rolling Analysis	410
10.8.2 Rolling of Tires	411
10.9 Uses of Low-Loss Materials	412
10.9.1 Timepieces	412
10.9.2 Frequency Stabilization and Control	413
10.9.3 Gravitational Measurements	413
10.9.4 Nanoscale Resonators	414
10.10 Impulses, Rebound, and Impact Absorption	414
10.10.1 Rationale	414
10.10.2 Analysis	415
10.10.3 Bumpers and Pads	418
10.10.4 Shoe Insoles, Athletic Tracks, and Glove Liners	419
10.10.5 Toughness of Materials	419
10.10.6 Tissue Viscoelasticity in Medical Diagnosis	420
10.11 Rebound of a Ball	421
10.11.1 Analysis	421
10.11.2 Applications in Sports	422
10.12 Applications of Soft Materials	424
10.12.1 Viscoelastic Gels in Surgery	424
10.12.2 Hand Strength Exerciser	424
10.12.3 Viscoelastic Toys	424
10.12.4 No-Slip Flooring, Mats, and Shoe Soles	425
10.13 Applications Involving Thermoviscoelasticity	425
10.14 Satellite Dynamics and Stability	426
10.15 Summary	428
10.16 Examples	429
10.17 Problems	431
Bibliography	431
<b>A: Appendix . . . . .</b>	<b>441</b>
A.1 Mathematical Preliminaries	441
A.1.1 Introduction	441
A.1.2 Functionals and Distributions	441
A.1.3 Heaviside Unit Step Function	442
A.1.4 Dirac Delta	442
A.1.5 Doublet	443
A.1.6 Gamma Function	445
A.1.7 Liebnitz Rule	445
A.2 Transforms	445
A.2.1 Laplace Transform	446

A.2.2 Fourier Transform	446
A.2.3 Hartley Transform	447
A.2.4 Hilbert Transform	447
A.3 Laplace Transform Properties	448
A.4 Convolutions	449
A.5 Interrelations in Elasticity Theory	451
A.6 Other Works on Viscoelasticity	451
Bibliography	452
<b>B: Symbols . . . . .</b>	<b>455</b>
B.1 Principal Symbols	455
<i>Index</i>	457

## Preface

This book is intended to be of use in a one-semester graduate course on the properties, analysis, and uses of viscoelastic materials. A precursor book, *Viscoelastic Solids*, has been used as a text in such a course. This book contains many updates, expanded coverage of the materials science of the causes of viscoelastic behavior, and of the properties of materials of biological origin, and applications of viscoelastic materials. The objective is to make the subject accessible and useful to students in a variety of disciplines in engineering and physical science. To that end, the coverage is intentionally broad. For research scientists and engineers or graduate students who pursue the subject via self-study, many references have been included to provide links to the literature. The subject may be profitably studied by undergraduate students, particularly those who are interested in vibration abatement, biomechanics, and the study of materials. Most of the book should be accessible to people who have completed an intermediate or an elementary course on the mechanics of deformable bodies. Exposure to elasticity theory, materials science, and vibration theory is helpful but not necessary.

A development of the theory is presented, including both transient and dynamic aspects, with emphasis on linear viscoelasticity. The structure of the theory is presented with the aim of developing physical insight. Methods for the solution of stress analysis problems in viscoelastic objects are developed and illustrated. Experimental methods for characterization of viscoelastic materials are explored in detail. Viscoelastic phenomena are described for a wide variety of materials, including polymers, metals, ceramics, geological materials, biological materials, synthetic composites, and cellular solids. High-damping alloys and composites are considered as well as materials that resist creep. To illustrate the sources of viscoelastic phenomena, we describe and analyze causal mechanisms, with cases of materials of extremely high damping and extremely low damping. The theory of viscoelastic composite materials is presented, with examples of various types of structures and the relationships between structure and mechanical properties. Many applications of viscoelasticity and viscoelastic materials are illustrated, with case studies and analysis of particular cases. Viscoelasticity is pertinent to applications as diverse as earplugs, gaskets, computer disks, satellite stability, medical diagnosis, injury

prevention, vibration abatement, tire performance, sports, spacecraft, and music. Examples of the use of viscoelastic materials in the prevention and alleviation of human suffering are explored. Supplementary material, including audio and video demonstrations of viscoelastic behavior, class resources, and research links, are provided on the following Website: <http://silver.neep.wisc.edu/~lakes>.

I thank the University of Wisconsin–Madison and the Vezzetti family for encouragement and support during a portion of the time that this book was in preparation. I am also grateful to my wife, Diana, for her patience and support. Finally, I thank Professor J. Giacomini and many students for their helpful suggestions.