Cambridge University Press
978-0-521-88568-3 - Viscoelastic Materials
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Excerpt
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Introduction: Phenomena

1.1 Viscoelastic Phenomena

Most solid materials are described, for small strains, by Hooke's law of linear elasticity: stress σ is proportional to strain ϵ. In one dimension, Hooke's law is as follows:

$$\sigma = E\epsilon, \tag{1.1}$$

with E as Young's modulus. Hooke's law for elastic materials can also be written in terms of a compliance J:

$$\epsilon = J\sigma. \tag{1.2}$$

Consequently, the elastic compliance J is the inverse of the modulus E:

$$J = \frac{1}{E}. \tag{1.3}$$

In contrast to elastic materials, a viscous fluid under shear stress obeys $\sigma = \eta d\epsilon/dt$, with η as the viscosity. In reality, all materials deviate from Hooke's law in various ways, for example, by exhibiting, both viscous-like and elastic characteristics. *Viscoelastic* materials are those for which the relationship between stress and strain depends on time (Figure 1.1). *Anelastic* solids represent a subset of viscoelastic materials: anelastic solids have a unique equilibrium configuration and ultimately recover fully after the removal of a transient load.

The stiffness and strength of materials is often illustrated by a stress–strain curve, which is obtained by applying a constant rate of strain to a bar of the material. If the material is linearly elastic, the curve is a straight line with a slope proportional to the elastic modulus (the heavy line in the right diagram in Figure 1.2). For a sufficiently large stress (the yield stress σ_y), the material exhibits yield as shown in Figure 1.2. This is a threshold phenomenon. A linearly viscoelastic material, by contrast, gives rise to a curved stress–strain plot (the left diagram in Figure 1.2) as demonstrated in §2.5. The reason for this rise is that during constant strain rate deformation, both time and strain increase together. The viscoelastic material is sensitive to time. Consequently, the curve on the left becomes steeper if the strain rate is increased. The residual strain eventually recovers to zero in a viscoelastic solid

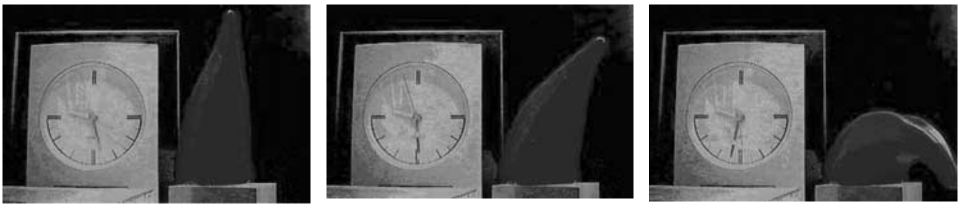

Figure 1.1. Droop with time (clock on left) of a viscoelastic pyramid.

(see below); a viscoelastic fluid undergoes a permanent residual strain. The elastic–plastic material is not sensitive to time or rate, but it has a threshold stress: the yield stress. There is always a residual strain ϵ_r after load removal if the yield stress has been exceeded. When testing and describing viscoelastic materials, it is preferable to apply a step strain or step stress in time rather than a ramp (constant rate of strain) because the effect of time is then isolated from any nonlinearity. The response to step strain is stress relaxation, and the response to step stress is creep.

Some phenomena in viscoelastic materials are:

(1) if the stress is held constant, the strain increases with time (creep);
(2) if the strain is held constant, the stress decreases with time (relaxation);
(3) the effective stiffness depends on the rate of application of the load;
(4) if cyclic loading is applied, hysteresis (a phase lag) occurs, leading to a dissipation of mechanical energy;
(5) acoustic waves experience attenuation;
(6) rebound of an object following an impact is less than 100 precent; and
(7) during rolling, frictional resistance occurs.

All materials exhibit some viscoelastic response. In common metals, such as steel or aluminum, as well as in quartz, at room temperature and at small strain, the behavior does not deviate much from the behavior of linearly elastic materials. Synthetic polymers, wood, and human tissue, as well as metals, at high temperature display large viscoelastic effects. In some applications, even a small viscoelastic response can be significant. To be complete, an analysis or design involving such materials must incorporate their viscoelastic behavior.

Knowledge of the viscoelastic response of a material is based on measurement. The mathematical formulation of viscoelasticity theory is presented in the following chapters with the aim of enabling prediction of the material response to arbitrary load histories. Even at present, it is not possible to calculate viscoelastic response

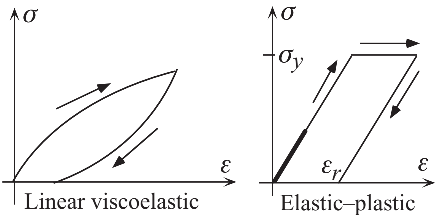

Figure 1.2. Stress–strain plots for deformation at constant strain rate followed by unloading. The plot on the left shows the behavior of a linearly viscoelastic material. The plot on the right shows the behavior of an ideal elastic–plastic material.

from models based on atomic and molecular models alone [1], although considerable understanding has evolved regarding causal mechanisms for viscoelastic behavior (see Chapter 8).

1.2 Motivations for Studying Viscoelasticity

The study of viscoelastic behavior is of interest in several contexts. First, materials used for structural applications of practical interest may exhibit viscoelastic behavior that has a profound influence on the performance of that material. Materials used in engineering applications may exhibit viscoelastic behavior as an unintentional side effect. In applications, one may deliberately make use of the viscoelasticity of certain materials in the design process, to achieve a particular goal. Second, the mathematics underlying viscoelasticity theory is of interest within the applied mathematics community. Third, viscoelasticity is of interest in some branches of materials science, metallurgy, and solid state physics because it is causally linked to a variety of microphysical processes and can be used as an experimental probe of those processes (Chapter 8). Fourth, the causal links between viscoelasticity and microstructure are exploited in the use of viscoelastic tests as an inspection tool as well as in the design of materials. Many applications of viscoelastic behavior are discussed in Chapter 10.

1.3 Transient Properties: Creep and Relaxation

1.3.1 Viscoelastic Functions $J(t)$, $E(t)$

Creep

Creep is a progressive deformation of a material under constant stress. In one dimension, suppose the history of stress σ as it depends on time t to be a step function with the magnitude σ_0, beginning at time zero:

$$\sigma(t) = \sigma_0 \mathcal{H}(t). \tag{1.4}$$

$\mathcal{H}(t)$ is the unit Heaviside step function defined as zero for t less than zero, one for t greater than zero, and $1/2$ for $t = 0$ (see Appendix §A.1.3). The strain $\epsilon(t)$ in a viscoelastic material will increase with time. The ratio,

$$J(t) = \frac{\epsilon(t)}{\sigma_0}, \tag{1.5}$$

is called the *creep compliance*. In linearly viscoelastic materials, the creep compliance is independent of stress level. The intercept of the creep curve on the strain axis is ascribed by some authors to instantaneous elasticity. However, no load can be physically applied instantaneously. If the loading curve is viewed as a mathematical step function, we remark that the region around zero time contains an infinite

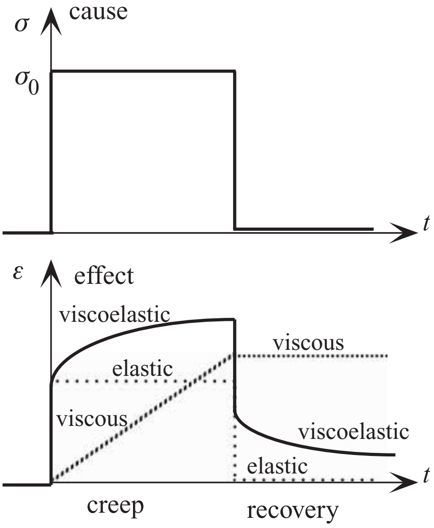

Figure 1.3. Creep and recovery. Stress σ and strain ϵ versus time t.

domain on a logarithmic scale, a topic we will return to later. If the load is released at a later time, the strain will exhibit recovery or a progressive decrease of deformation. Strain in recovery may or may not approach zero, depending on the material. The recovery phase is not included in Equations 1.4 and 1.5 but will be treated in §2.2.

The creep response in Figure 1.3 is shown beginning at the same time as the stress history, which is the *cause*. The corresponding functional form is $J(t) = j(t)\mathcal{H}(t)$, with $j(t)$ as a function defined over the entire time scale. This functional

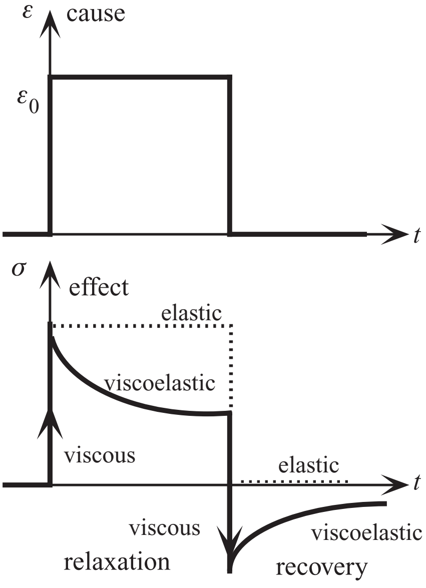

Figure 1.4. Relaxation and recovery.

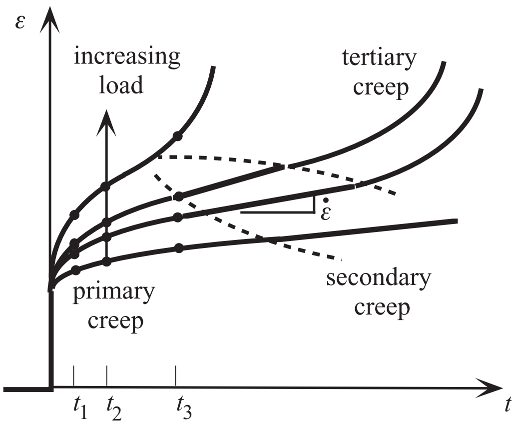

Figure 1.5. Regions of creep behavior. Strain ϵ versus time t, for different load levels.

form for $J(t)$ follows from the physical concept of causality, that is, the effect does not precede the cause.

Creep curves may exhibit three regions (Figure 1.5), *primary creep* in which the curve is concave down, *secondary creep* in which deformation is proportional to time, and *tertiary creep* in which deformation accelerates until creep rupture occurs. Tertiary creep is always a manifestation of nonlinear viscoelasticity, and secondary creep is usually nonlinear as well. Although secondary creep is represented by a straight line in a plot of strain versus time (constant strain rate), that straight line has nothing whatever to do with linear viscoelasticity. Linear response involves a linear relationship between cause and effect: stress and strain at a given time in the case of creep. Specifically, data taken at different load levels may be compared by considering isochronals or data at the same time. Data points at times t_1, t_2, and t_3 are illustrated in Figure 1.5. If the plot of stress versus strain at constant time is a straight line, the material may be linear. Secondary creep almost always entails nonlinear viscoelasticity. The nature of linear viscoelasticity and the distinction between linear and nonlinear behavior are presented in detail in §2.12 and §6.2.

Plotting Creep Results

Has the creep leveled off? From the top graph in Figure 1.6 (0 to 10 seconds), one might surmise that the creep strain is leveling off and is approaching an asymptotic value. The data used here extend over a wider range of time than shown in the top graph; the plot of the same data in the bottom graph on a scale 0 to 1,000 seconds shows that the creep has not leveled off but continues to progressively longer times. One might also surmise that there is an instantaneous elasticity corresponding to the intercept at time zero. However, use of a linear time scale fails to show processes that occur at short times. Comparing the two plots, it is evident that there is much creep occurring near time zero. The data used in the plots are from a power law: $\epsilon(t) = 10^{-3}t^{1/6}$. Such creep is representative of some experimental results gathered

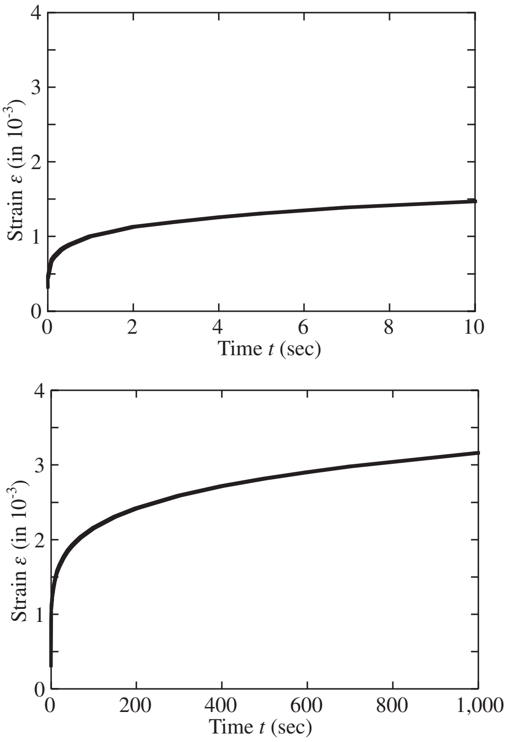

Figure 1.6. Creep on different time scales for the same creep strain $\epsilon(t) = 10^{-3}t^{1/6}$.

over a broad range of time. Therefore, it is customary to plot creep and relaxation results versus log time.

Relaxation

Stress relaxation is the gradual decrease of stress when the material is held at constant strain. If we suppose the strain history to be a step function of magnitude ϵ_0 beginning at time zero: $\epsilon(t) = \epsilon_0\mathcal{H}(t)$, the stress $\sigma(t)$ in a viscoelastic material will decrease as shown in Figure 1.4. The ratio,

$$E(t) = \frac{\sigma(t)}{\epsilon_0}, \tag{1.6}$$

is called the *relaxation modulus*. In linear materials, it is independent of strain level, so $E(t)$ is a function of time alone. The symbol E for Young's modulus as stiffness in uniaxial tension and compression is used in subsequent sections because the introductory presentations are restricted to one dimension.

Creep and relaxation can occur in shear or in volumetric deformation as well. The relaxation function for shear stress is called $G(t)$. For volumetric deformation, the elastic bulk modulus is called B (also called K). A corresponding relaxation function $B(t)$ may be defined as above, but with the stress as a hydrostatic stress. A similar distinction is made in the creep compliances, $J_G(t)$ for creep in shear, $J_E(t)$ for creep in extension, and $J_B(t)$ for creep in volumetric deformation.

The relaxation curve is drawn as decreasing with time, and the creep curve is drawn as increasing with time; it is natural to ask whether this must be so. To address this question, let us consider only passive materials and perform a thought experiment. A *passive* material is one without any external sources of energy; for a passive mechanical system, the only energy stored in the material is strain energy, and in a dynamical system, kinetic energy as well. An analytical definition of a passive material is given in §2.3. Let an initially unstrained specimen be deformed in creep under dead-weight loading. A material that raises the weight can do so only by performing positive work on the weight; this is impossible in a passive material. So for passive materials $J(t)$ is an increasing function.

A distinction may be made between aging and nonaging materials: in aging materials, properties change with time, typically time as measured following the formation or transformation of the material. Concrete, for example, is an aging material. The discussion here is, for the most part, restricted to nonaging materials.

1.3.2 Solids and Liquids

Elastic solids constitute a special case for which the creep compliance is $J(t) = J_0\mathcal{H}(t)$, with J_0 as a constant, which is the elastic compliance. Elastic materials exhibit immediate recovery to zero strain following release of the load. Viscoelastic materials that exhibit complete recovery after sufficient time following creep or relaxation are called *anelastic materials*. Viscous fluids constitute another special case in which the creep compliance is $J(t) = (1/\eta)t\mathcal{H}(t)$, with η as the viscosity. Creep deformation in viscous materials is unbounded.

In the modulus formulation, a *viscoelastic solid* is a material for which $E(t)$ tends to a finite, nonzero limit as time t increases to infinity; in a viscoelastic liquid, $E(t)$ tends to zero. In the compliance formulation, a viscoelastic solid is a material for which $J(t)$ tends to a finite limit as time t increases to infinity; in a viscoelastic fluid, $J(t)$ increases without bound as t increases.

The time scale extends from zero to infinity. In practice, creep or relaxation procedures in certain regions of the time scale are difficult to accomplish. For example, the region 10^{-10} sec to 0.01 sec is effectively inaccessible to most kinds of transient experiment because the load can be applied only so suddenly. Observation of the behavior of materials at longer times is limited by the patience and ultimately by the lifetime of the experimenter. In this vein, one may define [2] the dimensionless Deborah number D:

$$D \equiv \frac{\text{time of creep or relaxation}}{\text{time of observation}}. \tag{1.7}$$

If D is large, we perceive a material to be a solid even if it ultimately relaxes to zero stress. The difficulty in discriminating solids from liquids is a result of the finite lifetime of the human experimenter. Longer-term observations are also possible, as described by the Biblical prophetess Deborah: The mountains flowed before the Lord [3]. The original language may be translated as "flowed" [2], but some

Table 1.1. *Material properties*

Empty space: Quantum vacuum
Gases
Viscous liquids
Viscoelastic materials
Elastic solids

translations give "quaked" [RSV] or "melted" [KJV]. The flow of mountains is extremely slow, so that they appear solid to human observers, but are observed to flow before God. There is also an intermediate time scale of interest to engineers: we may wish that the materials used in structures behave as solids over the lifespan of human civilizations, which is longer than that of an individual. Geologists infer behavior of flowing rock over a time scale longer than that of human civilizations. We will return to these topics in discussions of experimental methods and of applications.

Viscoelastic materials are considered in the broader context of physical properties as follows (Table 1.1). Elastic solids support both shear stress and hydrostatic stress and their properties are independent of time or frequency. Viscoelastic materials exhibit time and frequency dependence. Viscous liquids support static hydrostatic stress; they generate shear stress only if the strain is changing with time. Gases are also viscous but they are orders of magnitude more compressible than liquids. Gases and liquids become indistinguishable at the critical point which corresponds to a particular temperature and pressure. In empty space, attractive force between conducting or dielectric surfaces is known as the *Casimir effect*. Isotropic elastic solids are describable by two elastic constants, for example, the shear and bulk modulus. Liquids and gases are also describable by two constants, the viscosity and the compressibility (inverse bulk modulus). By contrast, viscoelastic materials require a *function* of time or frequency to describe the behavior. Therefore, a rich set of physical phenomena can occur.

1.4 Dynamic Response to Sinusoidal Load: E^*, $\tan\delta$

Suppose the stress $\sigma(t)$ is varying (Figure 1.7) sinusoidally in time t, as follows:

$$\sigma(t) = \sigma_0 \sin(2\pi\nu t). \tag{1.8}$$

The frequency (in cycles per second or Hertz, abbreviated Hz) is called ν or f. The strain response of a linearly viscoelastic material is also sinusoidal in time, but the response will lag the stress by a phase angle δ

$$\epsilon(t) = \epsilon_0 \sin(2\pi\nu t - \delta). \tag{1.9}$$

The period T of the waveform is the time required for one cycle: $T = 1/\nu$. The phase angle is related to the time lag Δt between the sinusoids by $\delta = 2\pi(\Delta t)/T$.

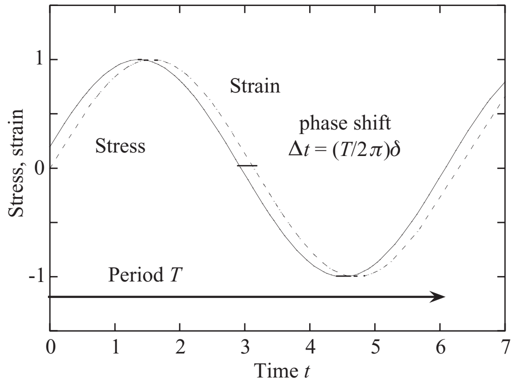

Figure 1.7. Stress and strain versus time t (in arbitrary units) in dynamic loading of a viscoelastic material.

To see this, the argument in Equation 1.9 may be written as

$$2\pi\nu t - \delta = 2\pi\nu t - \frac{2\pi\nu\delta}{2\pi\nu} = 2\pi\nu\left(t - \frac{\delta}{2\pi\nu}\right) = 2\pi\nu(t - \Delta t). \tag{1.10}$$

Therefore,

$$\Delta t = \frac{\delta}{2\pi\nu}, \tag{1.11}$$

with frequency as the inverse of period,

$$\nu = \frac{1}{T}$$

$$\delta = \frac{2\pi\,\Delta t}{T}. \tag{1.12}$$

As a result of the phase lag between stress and strain, the dynamic stiffness can be treated as a complex number E^*. *"Dynamic," in this context, refers to oscillatory input, not to any inertial effects:*

$$\frac{\sigma}{\epsilon_0} = E^* = E' + iE''. \tag{1.13}$$

The single and double primes designate the real and imaginary parts; they do not represent derivatives; $i = \sqrt{-1}$. The loss angle δ is a dimensionless measure of the viscoelastic damping of the material. The dynamic functions E', E'', and δ depend on frequency. The tangent of the loss angle is called the *loss tangent*: tanδ. In an elastic solid, tan$\delta = 0$. The relationship between the transient properties $E(t)$ and $J(t)$ and the dynamic properties E', E'', and tanδ is developed in §3.2.2. Dynamic viscoelastic behavior, in particular tanδ and its consequences, is at times referred to as *internal friction* or as *mechanical damping*.

1.5 Demonstration of Viscoelastic Behavior

Several commonly available materials may be used to demonstrate viscoelastic behavior. For example, Silly Putty®, sold as a toy, may be formed into a long rod and hung from a support so that it is loaded under its own weight. It will creep without limit, behaving as a liquid. It will also bounce like a rubber ball, behaving nearly elastically at a sufficiently high strain rate. Another example is a foam used for earplugs [4]. This foam can be compressed substantially and will recover most of the deformation in a period of a minute or so; simple creep experiments and demonstrations can also be performed with this material. As for the rate of decay of vibration, an aluminum tuning fork can be used to demonstrate the free decay of vibration. Following an impact, the fork is audible for many seconds, hence for thousands of cycles, demonstrating the low loss tangent of aluminum. A similarly shaped fork made of a material, such as a stiff plastic or wood (or a plastic ruler mounted as a cantilever and set into vibration) with a higher $\tan\delta$, will damp out its vibrations much more quickly. Sounds and waveforms for tuning forks of various materials can be found on the Internet at `http://silver.neep.wisc.edu/~lakes/Demo.html`.

1.6 Historical Aspects

A scientific awareness of viscoelastic behavior dates at least to the late eighteenth century. Coulomb [5] (1736–1806) reported studies of the torsional stiffness of wires by a torsional vibration method [6]. He also discussed damping of vibration and demonstrated experimentally that its principal cause was not air resistance but was a characteristic of the wire. As for creep and relaxation [7], Vicat in 1834 [8] surveyed the sagging of wires and of suspension bridges. Weber [9] and Kohlrausch [10] found deviations from perfect elasticity in galvanometer suspensions. Upon the release of torque on the galvanometer suspension, the instrument did not return to zero immediately; instead it converged gradually. This creep recovery was referred to as the *'elastic after effect'*; see also Zener [11]. Creep behavior was observed in silk threads under load in 1841 [12].

Viscoelastic behavior has been studied by eminent figures, such as Boltzmann, Coriolis, Gauss, and Maxwell [7]. Early mathematical modeling of relaxation processes included a stretched exponential formalism (discussed in Chapter 2), which was used to model creep in silk, glass fibers, and rubber. Maxwell [13] developed a relaxation analysis of gas viscosity that is also applicable to viscoelasticity. The integral representation of Boltzmann [14] for the stress–strain relationship forms the basis of the linear theory of viscoelasticity as it is currently understood [15]. The theory of integral equations and of functional analysis as developed by Volterra [16] provides much of the mathematical underpinning of viscoelastic behavior.

As polymeric materials assumed technological importance, intensive study of viscoelasticity began in the 1930s. Leaderman first suggested that an increase in