

## THE KERR SPACETIME

### Rotating Black Holes in General Relativity

Rotating black holes, as described by the Kerr spacetime, are the key to understanding the most violent and energetic phenomena in the Universe, from the core collapse of massive supernova explosions producing powerful bursts of gamma rays, to supermassive black hole engines that power quasars and other active galactic nuclei.

This book is a unique, comprehensive overview of the Kerr spacetime, with original contributions and historical accounts from researchers who have pioneered the theory and observation of black holes, and Roy Kerr's own description of his 1963 discovery. It covers all aspects of rotating black holes, from mathematical relativity to astrophysical applications and observations, and current theoretical frontiers. This book provides an excellent introduction and survey of the Kerr spacetime for researchers and graduate students across the spectrum of observational and theoretical astrophysics, general relativity and high-energy physics.

DAVID WILTSHIRE is a Senior Lecturer in the Department of Physics and Astronomy at the University of Canterbury, New Zealand. His research has spanned many areas in general relativity and cosmology, including black holes in higher dimensional gravity, brane worlds, quantum cosmology, dark energy and the averaging problem in inhomogeneous cosmology.

MATT VISSER is Professor of Mathematics at the Victoria University of Wellington, New Zealand. He has published widely in the areas of general relativity, quantum field theory, and theoretical cosmology. He is best known for his contributions to the theory of traversable wormholes, chronology protection, and analogue spacetimes.

SUSAN SCOTT is Associate Professor in the Centre for Gravitational Physics at the Australian National University. She is well known for her contributions to mathematical relativity and cosmology, and is currently President of the Australasian Society for General Relativity and Gravitation.

# THE KERR SPACETIME

## Rotating Black Holes in General Relativity

*Edited by*

DAVID L. WILTSHIRE

*University of Canterbury, Christchurch*

MATT VISSER

*Victoria University of Wellington*

SUSAN M. SCOTT

*Australian National University, Canberra*



CAMBRIDGE  
UNIVERSITY PRESS

**CAMBRIDGE**  
 UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314-321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi - 110025, India

103 Penang Road, #05-06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

[www.cambridge.org](http://www.cambridge.org)

Information on this title: [www.cambridge.org/9780521885126](http://www.cambridge.org/9780521885126)

© Cambridge University Press 2009

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 2009

*A catalogue record for this publication is available from the British Library*

*Library of Congress Cataloging in Publication data*

The Kerr spacetime : rotating black holes in general relativity / edited by David L. Wiltshire, Matt Visser, Susan Scott.

p. cm.

Includes index.

ISBN 978-0-521-88512-6

1. Kerr black holes. 2. Black holes (Astronomy) – Mathematical models. 3. Space and time – Mathematical models. 4. Kerr, R. P. (Roy P.) I. Wiltshire, David L., 1956– II. Visser, Matt. III. Scott, Susan, 1956– IV. Title.

QB843.B55K47 2009

523.8'875 – dc22 2008045980

ISBN 978-0-521-88512-6 Hardback

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party internet websites referred to in this publication, and does not guarantee that any content on such websites is, or will remain, accurate or appropriate.

## Contents

<i>Contributors</i>	<i>page</i> vii
<i>Foreword</i>	ix
<i>Preface</i>	xi
<i>List of Illustrations</i>	xiv
<b>I General relativity: Classical studies of the Kerr geometry</b>	
1 The Kerr spacetime – a brief introduction	3
<i>Matt Visser</i>	
2 The Kerr and Kerr–Schild metrics	38
<i>Roy P. Kerr</i>	
3 Roy Kerr and twistor theory	73
<i>Roger Penrose</i>	
4 Global and local problems solved by the Kerr metric	95
<i>Brandon Carter</i>	
5 Four decades of black hole uniqueness theorems	115
<i>David C. Robinson</i>	
6 Ray-traced visualisations in asymptotically flat spacetimes: the Kerr–Newman black hole	144
<i>Benjamin R. Lewis and Susan M. Scott</i>	
<b>II Astrophysics: The ongoing observational revolution in our understanding of rotating black holes</b>	
7 The ergosphere and dyadosphere of the Kerr black hole	161
<i>Remo Ruffini</i>	
8 Supermassive black holes	213
<i>Fulvio Melia</i>	
9 The X-ray spectra of accreting Kerr black holes	236
<i>Andrew C. Fabian and Giovanni Miniutti</i>	
10 Cosmological flashes from rotating black holes	281
<i>Maurice H. P. M. van Putten</i>	

**III Quantum gravity: Rotating black holes at the theoretical frontiers**

11 Horizon constraints and black hole entropy 311

*Steve Carlip*

12 Higher dimensional generalizations of the Kerr black hole 332

*Gary T. Horowitz*

**IV Appendices**

13 Gravitational field of a spinning mass as an example of algebraically special metrics 347

*Roy P. Kerr*

14 Gravitational collapse and rotation 349

*Roy P. Kerr*

*Index* 354

## Contributors

**Steve Carlip**

*Physics Department, University of California at Davis, CA 95616, USA*

**Brandon Carter**

*Observatoire de Paris–Meudon, Place Jules Janssen, F-92195 Meudon, France*

**Andrew C. Fabian**

*Institute of Astronomy, University of Cambridge, Madingley Road, Cambridge  
CB3 0HA, UK*

**Gary T. Horowitz**

*Physics Department, University of California at Santa Barbara, CA 93107, USA*

**Roy P. Kerr**

*Department of Physics and Astronomy and Department of Mathematics and  
Statistics, University of Canterbury, Private Bag 4180, Christchurch 8140,  
New Zealand*

**Benjamin R. Lewis**

*Centre for Gravitational Physics, Department of Physics, Australian National  
University, Canberra ACT 0200, Australia*

**Fulvio Melia**

*Department of Physics and Steward Observatory, University of Arizona, Tucson,  
Arizona 85721, USA*

**Giovanni Miniutti**

*Institute of Astronomy, University of Cambridge, Madingley Road, Cambridge  
CB3 0HA, UK*

**Roger Penrose**

*Mathematical Institute, Oxford University, 24-29 St Giles St, Oxford OX1 3LB, UK*

**Maurice H. P. M. van Putten**

*Kavli Institute for Astrophysics and Space Research, Massachusetts Institute of Technology 77 Massachusetts Avenue, 37-287, Cambridge, MA 02139, USA*

**David C. Robinson**

*Mathematics Department, King's College London, Strand, London WC2R 2LS, UK*

**Remo Ruffini**

*Dipartimento di Fisica, Università di Roma "La Sapienza", Piazzale Aldo Moro 5, I-00185 Roma, Italy; and ICRANet, Piazzale della Repubblica 10, I-65122 Pescara, Italy*

**Susan M. Scott**

*Centre for Gravitational Physics, Department of Physics, Australian National University, Canberra ACT 0200, Australia*

**Matt Visser**

*School of Mathematics, Statistics, and Computer Science, Victoria University of Wellington, PO Box 600, Wellington, New Zealand*

## Foreword

It is an amazing fact that there are roughly  $10^{20}$  rotating black holes in the observable universe, and that the spacetime near each of them is, to a very good approximation, given by an exact solution of Einstein's vacuum field equations discovered in 1963 by Roy Kerr. The Kerr spacetime is a defining feature of modern astrophysics. It is becoming increasingly important as the basis for understanding astrophysical processes from core collapse supernovae which produce gamma-ray bursts while forming black holes to the supermassive black hole engines that power quasars. These processes are the most violent and energetic phenomena in the universe since the Big Bang, and the key to understanding them is the Kerr geometry.

Yet the man behind this remarkable solution has remained somewhat enigmatic. When one of the contributors to this book heard that a conference was to be held to celebrate Roy's 70th birthday in 2004, he was surprised to learn that Roy was still alive! The idea of such a conference was suggested to me by another of my former teachers, Brian Wybourne, shortly before Brian passed away late in 2003. This book, which contains the invited lectures of the 2004 *Kerr Fest*, is the lasting result.

As someone who took courses from Roy at the University of Canterbury, one personal anecdote is in order. Roy amused his students with his laid back style and laconic humour, while always impressing upon us how to see quickly to the point in complex calculations, to back-track and fix our mistakes. He was also disdainful of needless bureaucracy, and took it as a badge of honour that he was the last person to deliver his exam scripts to the university Registry for printing. "I can't see why the Registry always want the exams so early," he complained. "In Texas we just used to write the exams on the board. Look, don't worry about the exam. I just rehash the old questions from past years."



In one lecture that year, after filling a blackboard with calculations that weren't getting closer to the expected solution, Roy wrote

$$\dots = \text{mess}$$

"Look, at this point I think I'd go off to the pub", he sighed. Standing back to look at the board for a minute, he enlightened us further. "Ah, I screwed up a factor of  $ct$  in the second line. . . I can't be bothered doing all that again. Just take down a note: *The stupid bastard screwed up the  $ct$ .*"

When we came to sit the exam for that course, some last minute rushing in its preparation became evident. The questions may just have been "rehashed", but when one put the values in, the algebra rapidly became very formidable, for question after question. One student left halfway through in an obvious state of distress. While wrestling with one particular question, which two pages of algebra made me suspect was actually not analytically soluble, the value of Roy's teaching became clear in a flash of inspiration. " $\dots = \text{mess}$ ", I wrote, "I can't be bothered solving this equation, but if I was to proceed here is the method I would use: . . ."

This anecdote not only illustrates the fun we had as Roy's students, but serves as a useful analogy for the state of understanding in 1963 about the problem of solving Einstein's equations for the exterior field of a rotating mass. After some decades of work the consensus of the experts was that the problem equalled a "mess", which most general relativists had given up on. The status of "black holes" as possible physical objects in the universe was also disputed. The term "black hole" itself had yet to be coined by Wheeler in 1967. Despite the work of Oppenheimer and Snyder on gravitational collapse in 1939, and the eventual understanding of the properties of the horizon in Schwarzschild's 1916 solution, everyone knew that realistic bodies rotated. Thus back in 1963 it was still a possibility that the Schwarzschild horizon was a mathematical curiosity, which might not survive the perturbation of adding angular momentum. Roy's solution changed all that. He established black holes as possible entities in the physical universe, and recent decades of many astrophysical observations have confirmed that nature has made good use of them.

Roy's brilliance in achieving what nobody had for decades, by pen and paper before the days of algebraic computing, was really a consequence of his insights into dealing with "messes". He understood deeply the importance of simplifying symmetry principles, of differential and integral constraints, and how to quickly sort out the relevant degrees of freedom from the irrelevant ones. The Kerr solution is a monumental legacy to Roy's incisive insight.

David Wiltshire

## Preface

The book is organized into three parts.

- *General relativity*. Wherein the contributors discuss the classical physics of rotating black holes. There is an orientational overview of the Kerr spacetime, followed by Roy Kerr's own account of its discovery, twistorial applications, constants of the motion, the uniqueness theorems and visualization of the Kerr geometry.
- *Astrophysics*. Observational evidence for rotating black holes, supermassive black holes, merger and collapse.
- *Quantum gravity*. Theoretical frontiers, black hole entropy and quantum physics, higher dimensional black strings and black rings.

The contributions encompass by and large the invited lectures of the *Kerr Fest: a Symposium on Black Holes in Astrophysics, General Relativity and Quantum Gravity*, held at Roy Kerr's home institution, the University of Canterbury, in Christchurch, New Zealand, in August 2004 to celebrate Roy's 70th birthday a few months beforehand on 16 May. As one of the original speakers, Zoltan Perjés, sadly passed away a few months after the Symposium, one of Roy's former colleagues who was unable to attend the Symposium, Roger Penrose, kindly agreed to submit an article in his place.

As appendices, we also reprint Roy Kerr's renowned *Physical Review Letter* of 1963, at less than two pages, stunning in its impact and brevity, along with his conference proceeding article presented at the *First Texas Symposium on Relativistic Astrophysics* in December 1963, in which the black hole property of the solution was first described.

There should be something in this book for almost everyone – there are relatively few books one can turn to to get significant coverage of the Kerr spacetime, and most of the relevant material is scattered around the scientific literature in small bits and pieces. We hope that this book will serve as a coherent starting point for

those interested in more technical details, and as a broad overview for those who are interested in the current state of play.

David Wiltshire

Matt Visser

Susan Scott

*Christchurch*

*Wellington*

*Canberra*

*January 2008*

Rotating Black Holes in General Relativity  
Cambridge University Press  
Edited by David L. Wilshire, Matt Visser, Susan M. Scott  
978-0-521-88512-6 The Kerr Spacetime  
Frontmatter

[More Information](#)



Roy Patrick Kerr, 2007.

## Illustrations

1.1	Rotating Kerr black hole.	<i>page 23</i>
1.2	Rotating Kerr black hole: Outer ergosurface and outer horizon.	31
1.3	Polar slice through a Kerr black hole.	33
2.1	Ivor Robinson and Andrzej Trautman.	41
2.2	Ezra T. Newman.	42
4.1	Conformal projection of $\{r, t\}$ section through a Kerr black hole.	108
4.2	Sketch of polar $\{r, \theta\}$ section through a Kerr black hole.	109
6.1	Multiple coordinate charts on the Schwarzschild spacetime.	145
6.2	Null geodesics in Kerr spacetime as visualized by GRworkbench.	146
6.3	Estimating the origin of a photon.	148
6.4	First-order effect of frame dragging.	152
6.5	Asymmetry of geodesic deflection in the Kerr spacetime.	154
6.6	Schwarzschild ray-tracing.	155
6.7	Kerr–Newman ray-tracing.	156
7.1	Cloud of particles corotating about an extremal spinning black hole.	162
7.2	Albert Einstein, Hideki Yukawa and John Archibald Wheeler.	164
7.3	Brandon Carter.	166
7.4	Potential in the equatorial plane of an extreme Kerr black hole.	167
7.5	Nuclear versus gravitational binding energy.	168
7.6	Extraterrestrial civilization as idealized by Roger Penrose.	170
7.7	The ergosphere as sketched by Ruffini & Wheeler.	172
7.8	Particle decay process in the field of a black hole.	174
7.9	Annotated first page of a preprint by Floyd and Penrose.	175
7.10	D. Christodoulou’s Ph.D. defense.	176
7.11	Introducing the black hole.	177
7.12	Riccardo Giacconi and Luigi Broglio; the Uhuru satellite.	181
7.13	Cygnus–X1.	183
7.14	Magnetic field lines in the equatorial plane of a Kerr black hole.	184
7.15	Lines of currents in the magnetosphere.	184
7.16	Grayscale representation of the image of Cygnus A at 5 GHz.	185
7.17	Location of the black hole Cygnus–X1.	186
7.18	A model of how the black hole created the bubble.	186

*List of illustrations*

xv

7.19	Zel'dovich after donating his book to Pope John Paul II.	187
7.20	<i>Vela 5A</i> and <i>5B</i> satellites; a typical event.	188
7.21	The CGRO satellite; observed GRBs in galactic coordinates.	190
7.22	Some GRB light curves observed by BATSE.	191
7.23	Energy fluence-averaged hardness ratio for short and long GRBs.	192
7.24	The Italian–Dutch satellite <i>BeppoSAX</i> .	193
7.25	“Dyado-torus” for an extreme Kerr–Newman black hole.	196
7.26	Expansion of the PEMB pulse.	199
7.27	The GRB afterglow phase.	200
7.28	Comparison between approximate and exact EQTSs.	202
7.29	GRB 991216 within our theoretical framework.	205
7.30	Energy radiated in the P-GRB and in the afterglow.	206
7.31	TEST (Traction of Events in Space and Time) by Attilio Pierelli.	207
7.32	The ICRA and ICRANet logos.	207
8.1	The quasar 3C 273.	214
8.2	HST image of the host galaxy containing the quasar QSO 1229+204.	215
8.3	Collision between two galaxies – unraveling of the spiral disks.	216
8.4	The <i>Chandra</i> Deep Field North X-ray image.	218
8.5	Composite image of Centaurus A.	220
8.6	VLA image of the powerful central engine in the nucleus of Cygnus A.	221
8.7	Black-hole-powered jet of plasma streaming from the center of M87.	222
8.8	Stars surrounding the supermassive black hole at the galactic center.	224
8.9	The butterfly-shaped galaxy NGC 6240.	227
8.10	The spiral galaxy NGC 2613.	228
8.11	Masses of supermassive black holes versus stellar velocity dispersion.	229
8.12	Black hole accretion rate versus redshift.	230
8.13	History of the accretion rate density.	231
8.14	Spectra for flux density and polarization from Sagittarius A*.	232
8.15	Polarization maps of Sagittarius A*.	234
9.1	X-ray and reflection spectra of an accreting black hole.	238
9.2	Photoionized gas near an AGN; typical spectrum of a Seyfert 2 galaxy.	240
9.3	Emission line modified by Doppler/gravitational shifts.	241
9.4	Inclination and emissivity effects on line profiles.	242
9.5	X-ray reflection spectrum.	244
9.6	Innermost stable circular orbit.	245
9.7	Fe line in the S0 spectrum of MCG–6-30-15.	247
9.8	Broadband light curve and broad Fe line.	248
9.9	HEG spectrum and Fe line.	250
9.10	Flux–flux plots.	252
9.11	Fe line flux and reflection fraction.	256
9.12	Typical emissivity and line profiles.	257
9.13	PLC photon index and RDC normalization.	257
9.14	Variations of the line profile; observed and predicted.	258
9.15	Light curves during a flare.	259
9.16	Spectra compared to power laws.	260
9.17	Spectral decomposition: variable power-law plus reflection component.	262

xvi	<i>List of illustrations</i>	
9.18	Flux–flux plots.	263
9.19	Prominent relativistic lines in GBHCs.	265
9.20	Line profile of XTE J1650–500; Fe line flux.	266
9.21	Model based on X-ray data of Seyfert 1 and NLS1 galaxies.	269
9.22	Constellation-X simulated spectrum.	270
9.23	Time-averaged line profile of NGC 3516.	273
9.24	Red feature folded light curve; on- and off-phases.	274
9.25	Excess emission image; observational and theoretical.	275
10.1	Redshift-corrected distributions of 27 long gamma ray bursts.	283
10.2	Black holes with small kick velocities.	289
10.3	Uniformly magnetized torus around a black hole.	290
10.4	Stability diagram: neutral stability curves.	295
10.5	GRB-supernovae from rotating black holes.	301
11.1	The Carter–Penrose diagram for a non-extremal BTZ black hole.	317
11.2	A spacelike “stretched horizon” $\Delta$ .	322