NEW DIRECTIONS IN LINEAR ACOUSTICS AND VIBRATION

The field of acoustics is of immense industrial and scientific importance. The subject is built on the foundations of linear acoustics, which is widely regarded as so mature that it is fully encapsulated in the physics texts of the 1950s. This view was changed by developments in physics such as the study of quantum chaos. Developments in physics throughout the last four decades, often equally applicable to both quantum and linear acoustic problems but overwhelmingly more often expressed in the language of the former, have explored this. There is a significant new amount of theory that can be used to address problems in linear acoustics and vibration, but only a small amount of reported work does so. This book is an attempt to bridge the gap between theoreticians and practitioners, as well as the gap between quantum and acoustic. Tutorial chapters provide introductions to each of the major aspects of the physical theory and are written using the appropriate terminology of the acoustical community. The book will act as a quick-start guide to the new methods while providing a wide-ranging introduction to the physical concepts.

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QUANTUM CHAOS, RANDOM MATRIX THEORY, AND COMPLEXITY

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CAMBRIDGE
UNIVERSITY PRESS

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Shaftesbury Road, Cambridge CB2 8EA, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India

103 Penang Road, #05–06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of Cambridge University Press & Assessment, a department of the University of Cambridge.

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www.cambridge.org
Information on this title: www.cambridge.org/9780521885089

First published 2010
First paperback edition 2015

A catalogue record for this publication is available from the British Library

Library of Congress Cataloging-in-Publication data
New directions in linear acoustics and vibration : quantum chaos, random matrix theory, and complexity / [edited by] Matthew Wright, Richard Weaver.
p. cm.
Includes bibliographical references and index.
ISBN 978-0-521-88508-9
1. Sound-waves. 2. Vibration. 3. Acoustical engineering. I. Wright, Matthew, 1966– II. Weaver, Richard, 1949– III. Title.
QC243.N49 2010
620.2–dc22 2010019825

ISBN 978-0-521-88508-9 Hardback
ISBN 978-1-107-51345-7 Paperback

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Foreword

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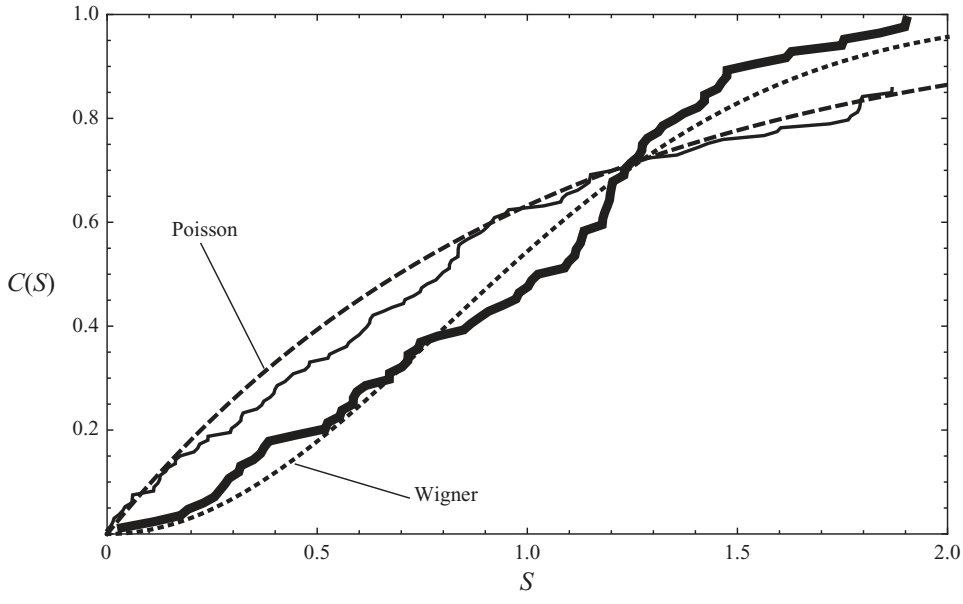
In the early 1970s, Martin Gutzwiller and Roger Balian and Claude Bloch described quantum spectra in terms of classical periodic orbits, and in the mid 1970s it became clear that the random matrix theory devised for nuclear physics would also describe the statistics of quantum energy levels in classically chaotic systems. It seemed obvious even then that these two great ideas would find application in acoustics, but it has taken more than three decades for this insight to be fully implemented. The chapters in this fine collection provide abundant demonstration of the continuing fertility, in the understanding of acoustic spectra, of periodic orbit theory and the statistical approach. The editors' kind invitation to me to write this foreword provides an opportunity to make a remark about each of these two themes.

First, here is a simple argument for periodic orbit theory being the uniquely appropriate tool for describing the acoustics of rooms. The reason for confining music and speech within auditoriums – at least in climates where there is no need to protect listeners from the weather – is to prevent sound from being attentuated by radiating into the open air. But if the confinement were perfect, that is, if the walls of the room were completely reflecting, sounds would reverberate forever and get confused. To avoid these extremes, the walls in a real room must be partially absorbing. This has the effect of converting the discrete eigenvalues with perfectly reflecting walls into resonances. I will argue that for real rooms the width of resonances usually exceeds their spacing. This is important because it casts doubt on the usefulness of the concept of an individual mode in assessing the acoustic response of rooms; a smoothed description of the spectrum seems preferable. But smoothing is precisely what periodic orbit theory naturally describes. When there is no absorption, the contributions from the long periodic orbits make the convergence of the sum problematic, frustrating the direct calculation of individual eigenvalues, for example, in quantum chaology. Absorption attentuates the long orbits, and the oscillatory contributions from few shortest orbits are sufficient to describe the acoustic response. But these few orbits are important: the crudest smoothing, based simply on the average spectral density, obliterates all the spectral oscillations and fails to capture the characteristics of most real rooms.

To assess the significance of absorption, start from the Weyl counting formula for the number N of modes with frequencies less than f, for a room of volume L^3:



if the speed of sound is $c = 330\,\mathrm{ms}^{-1}$,

$$N = \frac{4\pi L^3 f^3}{3c^3}.$$

In the presence of absorption, modeled approximately by an exponential amplitude decay time T, that is, intensity $\sim \exp(-2t/T)$, the resonance width corresponds to a frequency broadening,

$$\Delta f = \frac{1}{2\pi T}.$$

Thus, incorporating the reverberation time T_{60}, corresponding to 60-dB intensity reduction, that is, $T = T_{60}/3\log_e 10$, the number ΔN of modes smoothed over by the broadening is

$$\Delta N = 6\log_e 10 \frac{L^3 f^2}{c^3 T_{60}}.$$

For estimates, we can choose the frequency middle A ($f = 440\,\mathrm{Hz}$). Then, for a small auditorium with $L = 6\,\mathrm{m}$, and a reverberation time $T_{60} = 0.7\,\mathrm{s}$, $\Delta N \sim 23$, which is unexpectedly large for such a small room. For the Albert Hall in London, where the effective $L \sim 60\,\mathrm{m}$, and taking $T_{60} = 2\,\mathrm{s}$, $\Delta N \sim 8{,}200$. These estimates strongly suggest that there is little sense in studying individual modes.

Second, here is an unusual application of spectral statistics from 1993, inspired by a visit to Loughborough University, where I talked about quantum chaos and mentioned that the ideas could be usefully applied in acoustics. Afterward, Robert Perrin showed me his measurements (Perrin et al. 1983) of eigenfrequencies of one English church bell, ranging from 292.72 Hz – the lowest mode, called the hum, through the first few harmonics, with their traditional names Fundamental, Tierce, Quint, Nominal, Twister, Superquint – up to the 134th frequency of 9,285 Hz. This

provided sufficient data to make a first attempt to understand the frequency spacings distribution.

I did this in two ways. First, taking the whole set of 134 frequencies, unfolding them by fitting the counting function (spectral staircase) to a cubic function, and then calculating the 133 spacings, normalized to unit mean. The resulting cumulative spacings distribution $C(S)$ = fraction of spacings less than S, fits the Poisson distribution $1 - \exp(-S)$ reasonably well (the thin and dashed curves in the figure). This is not surprising because the bell has approximate rotation symmetry, and the whole set of frequencies conflates subsets with different numbers l of nodal meridians ("angular momentum quantum number"). Fortunately the value of l for each frequency was given; l ranged from 0 to 28, but only the subsets with $0 \leq l \leq 10$ included sufficient frequencies to generate sensible statistics. In the second procedure, I unfolded these subsets separately and conflated the spacings afterwards, thereby generating the heavy curve in the figure. This is better fitted to the Wigner cumulative distribution $1 - \exp(-S^2/4)$ (the dotted curve in the figure), indicating strong repulsion of neighboring frequencies in each l-subset. The precise fit is not important because the Wigner distribution should apply when the ray geodesics on the bell – "classical paths" – are chaotic, whereas the vibrations of the bell, regarded as a thin elastic sheet, are probably integrable, with frequencies given by the modes of a one-dimensional "radial" equation, albeit of fourth order.

Reference

Perrin, R., Charnley, T. & DePont, L. (1983), "Normal modes of the modern English church bell," *J. Sound. Vib.* **90**, 29–49.