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978-0-521-88389-4 - Combinatorics: The Rota Way

Joseph P. S. Kung, Gian-Carlo Rota and Catherine H. Yan

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