

Contents

<i>Preface</i>	<i>page ix</i>
1 Sets, Functions, and Relations	1
1.1. Sets, Valuations, and Boolean Algebras	1
1.2. Partially Ordered Sets	9
1.3. Lattices	17
1.4. Functions, Partitions, and Entropy	28
1.5. Relations	44
1.6. Further Reading	52
2 Matching Theory	53
2.1. What Is Matching Theory?	53
2.2. The Marriage Theorem	54
2.3. Free and Incidence Matrices	62
2.4. Submodular Functions and Independent Matchings	67
2.5. Rado's Theorem on Subrelations	74
2.6. Doubly Stochastic Matrices	78
2.7. The Gale-Ryser Theorem	94
2.8. Matching Theory in Higher Dimensions	101
2.9. Further Reading	105
3 Partially Ordered Sets and Lattices	106
3.1. Möbius Functions	106
3.2. Chains and Antichains	126
3.3. Sperner Theory	136
3.4. Modular and Linear Lattices	147
3.5. Finite Modular and Geometric Lattices	161
3.6. Valuation Rings and Möbius Algebras	171
3.7. Further Reading	176

4	Generating Functions and the Umbral Calculus	178
4.1.	Generating Functions	178
4.2.	Elementary Umbral Calculus	185
4.3.	Polynomial Sequences of Binomial Type	188
4.4.	Sheffer Sequences	205
4.5.	Umbral Composition and Connection Matrices	211
4.6.	The Riemann Zeta Function	218
5	Symmetric Functions and Baxter Algebras	222
5.1.	Symmetric Functions	222
5.2.	Distribution, Occupancy, and the Partition Lattice	225
5.3.	Enumeration Under a Group Action	235
5.4.	Baxter Operators	242
5.5.	Free Baxter Algebras	246
5.6.	Identities in Baxter Algebras	253
5.7.	Symmetric Functions Over Finite Fields	259
5.8.	Historical Remarks and Further Reading	270
6	Determinants, Matrices, and Polynomials	272
6.1.	Polynomials	272
6.2.	Apolarity	278
6.3.	Grace's Theorem	283
6.4.	Multiplier Sequences	291
6.5.	Totally Positive Matrices	296
6.6.	Exterior Algebras and Compound Matrices	303
6.7.	Eigenvalues of Totally Positive Matrices	311
6.8.	Variation Decreasing Matrices	314
6.9.	Pólya Frequency Sequences	317
	Selected Solutions	324
	<i>Bibliography</i>	369
	<i>Index</i>	389