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978-0-521-88336-8 - Harmonic Analysis on Finite Groups: Representation Theory,
Gelfand Pairs and Markov Chains

Tullio Ceccherini-Silberstein, Fabio Scarabotti and Filippo Tolli

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