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978-0-521-88300-9 — *J*-Contractive Matrix Valued Functions and Related Topics
Damir Z. Arov , Harry Dym
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J-Contractive Matrix Valued Functions and
Related Topics

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Dedicated to our wives Natasha and Irene, for their continued support and encouragement, and for being ideal companions on the path of life.

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Preface

This book is one of the products of the joint work of the authors over the last 15 years. We first met at the IWOTA and MTNS conferences in Japan in 1991. At the time one of us knew very little English and the other's Russian was limited to *da* and *net*. Fortunately, our mutual friend and colleague Israel Gohberg was able and ready to act as an interpreter, and tentative arrangements were made for the first author to visit the second at the Weizmann Institute. These visits were repeated each year for three or more months, and a research program that focused on direct and inverse problems for canonical systems of integral and differential equations was initiated. This program made extensive use of the existing theory of J -contractive and J -inner mvf's (matrix valued functions) as well as requiring new developments. This monograph is a comprehensive introduction to that theory, and a number of its applications. Much of it is either new, or not conveniently accessible. A second volume on canonical systems of integral and differential equations is planned.

The authors gratefully acknowledge and thank: Victor Katsnelson for supplying helpful remarks and a number of references; Ruby Musrie, Diana Mandelik, Linda Alman and Terry Debesh for typing assorted sections; Clare Lendrem for her careful copy editing of a not so final draft; the administration of South Ukrainian Pedagogical University for authorizing extended leaves of absence to enable the first author to visit the second and finally, and most importantly, the Minerva Foundation, the Israel Science Foundation, the Arthur and Rochelle Belfer Institute of Mathematics and Computer Science, and the Visiting Professorship program at the Weizmann Institute for the financial support that made these visits possible and enabled the authors to work together under ideal conditions.

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